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PARTITION OF MEASURABLE SETS

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Abstract

The theory of vector measure has attracted much interest among researchers in the recent past. Available results show that measurability concepts of the Lebesgue measure have been used to partition subsets of the real line into disjoint sets of finite measure. In this paper we partition measurable sets in $\Re^n$ for $n \geq 3$ into disjoint sets of finite dimension.

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1 INTRODUCTION

In this article, measurable cover estimate technique is applied to partition measurable sets. The utility of properties such as extension, countable additivity and contraction of the projective tensor product of vector measure duality is demonstrated. The process of partition requires that we vary all countable coverings of sets in a ring by sets in a σ-ring. Sequences of monotonically increasing (or decreasing) sets and integrability concepts of strictly increasing (or decreasing) functions are considered. For notations and basic concepts used in this paper, reference may be made to [1, 2, 3, 4, 5, 6]

2 Basic Concepts

2.1 Projective tensor product vector measure duality

Let $X_1,......,X_n$ and Z be real Banach spaces with $\Phi:\Pi_{i=1}^n X_i \to Z$ being a continuous linear function. If $\mu_1: R_1 \to X_1$,........, $\mu_n: R_n \to X_n$ are countably additive vector measures, then the product $\Pi_{i=1}^n \mu_i$ is a countably additive vector measure defined on the ring $\Pi_{i=1}^n R_i$ generated by sets of the form $E_1 \times .... \times E_n$. Let $(G(R_1),......,G(R_n))$ be a set of σ - rings generated by rings $R_1,.......,R_n$ respectively. If the extension of the vector measure $\Pi_{i=1}^n \mu_i : \Pi_{i=1}^n R_i \to \Pi_{i=1}^n X_i$ to a vector measure $\Pi_{i=1}^n \mu_i^* : \Pi_{i=1}^n G(R_i) \to \Pi_{i=1}^n X_i$ coincide with respect to a linear function $\Phi:\Pi_{i=1}^n X_i \to Z$, where $Z = \Phi(\mu_1^*(E_1),.........,\mu_n^*(E_n))$ for $\mu_i^*(E_i) \in X_i$, $1 \le i \le n$ and $\Pi_{i=1}^n X_i$ is a Banach space, then $(\Pi_{i=1}^n)_\Phi < \mu_i^*(E_i), Z' >$ is called the projective tensor product of vector measure duality between Z and its dual space Z' .

2.2 3 D section of a measurable set

Fix p_{n-2} for $n \in \aleph$ and $n \ge 3$. The set $(\Pi_{i=1}^n E_i)^{p_{n-2}} = E_{n-2} \times E_{n-1} \times E_n$ is called a three dimensional section of a measurable set $\Pi_{i=1}^n E_i = N(f)$ such that $\iiint f \ \delta(\Pi_{i=n-2}^n)_\Phi < \mu_i^*(E_i), Z' > < \infty$.

3 Results

Proposition 1 Let $f:\Pi_{i=n-2}^n G(R_i) \to \Pi_{i=n-2}^n X_i$ be an integrable function with respect to $\Pi_{i=n-2}^n < \mu_i^*, Z' >$. Suppose the set $((e_{n-2},e_{n-1},e_n): f(e_{n-2},e_{n-1},e_n) > P)$ for $P > 0$, is measurably covered by N(f). If $(\Pi_{i=n-2}^n E_i)_{e_{n-2}}$ is a countable partition of a set $((e_{n-2},e_{n-1},e_n): f(e_{n-2},e_{n-1},e_n) > P)_{e_{n-2}}$ for a fixed $e_{n-2} \in E_{n-2}$ such that $\iint f \ \delta(\Pi_{i=n-1}^n)_\Phi < \mu_i^*(E_i), Z' > < M$ for $M > 0$, then $< T_{(\Pi_{i=n-1}^n)_\Phi \mu_i^*(E_i)}(f), Z' >= 0$

Proof. Let $N(f) = ((e_{n-2},e_{n-1},e_n): f(e_{n-2},e_{n-1},e_n) \ne 0)$. It can be deduced from [2, p.18] that $(\Pi_{i=n-2}^n E_i)_{e_{n-2}} = \Pi_{i=n-1}^n E_i$ for a fixed $e_{n-2} \in E_{n-2}$. Since by hypothesis $((e_{n-2},e_{n-1},e_n): f(e_{n-2},e_{n-1},e_n) > P)$ $\Theta \ N(f)$ and $(\Pi_{i=n-2}^n E_i)_{e_{n-2}}$ is a countable partition of $((e_{n-2},e_{n-1},e_n): f(e_{n-2},e_{n-1},e_n) > P)_{e_{n-2}}$, it follows that $(\Pi_{i=n-2}^n E_i)_{e_{n-2}} = \Pi_{i=n-1}^n E_i \Theta (N(f))_{e_{n-2}} \Rightarrow ((e_{n-2,} \ e_{n-1},e_n): f(e_{n-2},e_{n-1},e_n) > P)_{e_{n-2}} = \cup_{k=1}^\infty \Pi_{i=n-1}^n E_{k_i}$ for a countable partition $\Pi_{i=n-1}^n E_{k_i}$ of $\Pi_{i=n-1}^n E_i$. In this case, $\Pi_{i=n-1}^n E_i$ can be written uniquely as a countable union of disjoint sets (see [3 p.5]). Now, $\cup_{k=1}^\infty \Pi_{i=n-1}^n E_{k_i} = \Pi_{i=n-1}^n E_i \Rightarrow P\chi_{\Pi_{i=n-1}^n E_i} \le \chi_{\Pi_{i=n-1}^n E_i} f$. Let $(N(f))_{e_{n-2}} = \Pi_{i=n-1}^n C_i$ be a set in a σ -ring $\Pi_{i=n-1}^n G(R_i)$ It follows that $\Pi_{i=n-1}^n E_i \subset \Pi_{i=n-1}^n C_i \Rightarrow \Pi_{i=n-1}^n E_i \Theta \Pi_{i=n-1}^n C_i$. Let $\mu_{i_{C_i}}^*$ be a vector measure defined on $G(R_i)$ for $i = n-1, n$. By extension procedures and integral representation of the contraction of $\mu_{n-1} \times \mu_n$ as illustrated in [6], we have $P < T_{(\Pi_{i=n-1}^n)_\Phi \mu_i^*(C_i \cap E_i)}(f), Z' > \le \iint f \ \delta(\Pi_{i=n-1}^n)_\Phi < \mu_{i_{C_i}}^*(E_i), Z' > < M \ \ P < T_{(\Pi_{i=n-1}^n)_\Phi \mu_i^*(C_i \cap E_i)}(f), Z' >$ $\le \iint f \ \delta(\Pi_{i=n-1}^n)_\Phi < \mu_i^*(C_i \cap E_i), Z' > < M \Rightarrow < T_{(\Pi_{i=n-1}^n)_\Phi \mu_i^*(C_i \cap E_i})(f), Z' > < M/P$ But $E_i \subset C_i$ for

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$1 = n-1,\ n$ .Therefore, $< T_{(\Pi_{i=n-1}^{n})_{\Phi}\mu_i^*(E_i)}(f), Z' >= M/P.$ As $P \to \infty$, $< T_{(\Pi_{i=n-1}^{n})_{\Phi}\mu_i^*(E_i)}(f), Z' >= 0$

Proposition 2 Let f and $f_n (n = 1,2,....)$ for $f_n \uparrow$ be integral functions with respect to $(\Pi_{i=n-2}^{n})_{\Phi} < \mu_i^*, Z' >$ such that $N(f_n)$ is measurably covered by $N(f)$. If $E_i = N(g_n) \subset N(f) - N(f_n)$ for $(n = 1,2,....)$ and $i = n-2,\ n-1,\ n$, we have $\iiint g_n\ \delta(\Pi_{i=n-2}^{n})_{\Phi} < \mu_i^*(E_i), Z' > < \varepsilon$, for $\varepsilon > 0$

Proof.Let $N(f_n) = ((e_{n-2}, e_{n-1}, e_n) : f_n(e_{n-1}, e_{n-1}, e_n) \neq 0)$ and $N(f) = ((e_{n-2}, e_{n-1}, e_n) : f(e_{n-2}, e_{n-1}, e_n) \neq 0)$ be measurable sets of finite measure.Since $N(g_n) \subset N(f) - N(f_n)$ for each n, it follows that $g_n \downarrow 0$ for each $n.$ Let $E_{n-2} = N(g_{n-2})$ and A_{k_i} be a countable partition of E_i for $i = n-2, n-1, n.$ Now, $A_{k_i} \uparrow E_i$ for $i = n-2, n-1, n$ (see [3 p.20]).Suppose $N(g_{n-1}) = E_i - \cup_{k=1}^{\infty} A_{k_i}$ for $i = n-2, n-1, n$ It follows that $N(g_{n-1})$ $\downarrow \varnothing$ Since g_{n-1} is integrable with respect to $\Pi_{i=n-1}^{n})_{\Phi} < \mu_i^*, Z' >$ (see [5]), $\Rightarrow \Sigma_{k=1}^{\infty} \iiint g_{n-1}$ $\delta(\Pi_{i=n-1}^{n})_{\Phi} < \mu_i^*(E_i - A_{k_i}), Z' >= 0$ Let us vary the partition $A_{k_{o_i}}^{j}$ of A_{k_i} such that $\Pi_{i=n-2}^{n} A_{k_i} = \cup_{j=1}^{\infty} \cup_{k_o=1}^{\infty} A_{k_{o_i}}^{j}$. It follows that $\Sigma_{j=1}^{\infty} \Sigma_{k_o=1}^{\infty} \iiint g_{n-1}\ \delta(\Pi_{i=n-1}^{n})_{\Phi} < \mu_i^*(E_i - A_{k_{o_i}}^{j}), Z' >< \varepsilon.$ Let $A_i = \cup_{j=1}^{\infty} \cup_{k_o=1}^{\infty} A_{k_{o_i}}^{j}$ $\Rightarrow \iiint g_{n-1}\delta\ (\Pi_{i=n-1}^{n})_{\Phi} < \mu_i^*(E_i - A_i), Z' >< \varepsilon.$ As shown in [3 , p. 21], if we partition E_i into disjoint sets $E_i - A_i$ and A_i for $i = n-2, n-1, n$ and apply the additivity property of multiple integral, we have $\iiint g_n\ \delta(\Pi_{i=n-2}^{n})_{\Phi} < \mu_i^*(E_i), Z' >= \iiint g_n \left| \delta(\Pi_{i=n-2}^{n})_{\Phi} < \mu_i^*(A_i), Z' > \right.$. Since A_i is a countable partition of E_i ,then $A_i \uparrow E_i$ and $A_i \subset E_i$ for each i.Since $E_i = N(g_n)$, $g_n \downarrow 0$ for $i = n-2,\ n-1,\ n$, and $n = 1,2,.... \Rightarrow \iiint g_n \delta(\Pi_{i=n-2}^{n})_{\Phi} < \mu_i^*(A_i), Z' >= 0,$ it follows that $\iiint g_n \delta(\Pi_{i=n-2}^{n})_{\Phi} < \mu_i^*(E_i), Z' >< \varepsilon$.

Proposition 3 Let $\Pi_{i=n-2}^{n} E_i$ be measurably covered by $N(f_n)$.If $\Pi_{i=n-2}^{n} C_i$ is a section of $\Pi_{i=n-2}^{n} E_i$ such that $C_{k_i} \downarrow \varnothing$ for every countable partition C_{k_i} of C_i for $i = n-2, n-1, n$, then

$$\iiint f_n \delta(\Pi_{i=n-2}^{n})_{\Phi} < \mu_i^*(E_i), Z' > \ = \iiint f_n \delta(\Pi_{i=n-2}^{n})_{\Phi} < \mu_i^*(E_i - C_i), Z' > \iiint f_n \delta(\Pi_{i=n-2}^{n})_{\Phi} < \mu_i^*(C_i - C_{k_i}), Z' >$$

Proof. Choose $\beta > 0$ such that $\Pi_{i=n-2}^{n} C_i = ((x_{n-2}, x_{n-1}, x_n) : f(x_{n-2}, x_{n-1}, x_n) > \beta).$ Since $\Pi_{i=n-2}^{n} C_i$ is a section of $\Pi_{i=n-2}^{n} E_i$, then $C_i \uparrow E_i$ for $i = n-2, n-1, n.$ Let $\iiint f_n \delta(\Pi_{i=n-2}^{n})_{\Phi} < \mu_i^*(E_i - C_i), Z' >< \varepsilon$ for all n.Suppose we partition each E_i into disjoint sets $E_i - C_i$ and C_i for $i = n-2, n-2, n$. Since $\Pi_{i=n-2}^{n} E_i \Theta N(f_n)$, it follows that $\chi_{\Pi_{i=n-2}^{n} E_i} f_n = \chi_{\Pi_{i=n-2}^{n}(E_i - C_i)} f_n + \chi_{\Pi_{i=n-2}^{n} C_i} f_n \Rightarrow \iiint f_n$ $\delta(\Pi_{i=n-2}^{n})_{\Phi} < \mu_i^*(E_i), Z' >= \iiint f_n \delta(\Pi_{i=n-2}^{n})_{\Phi} < \mu_i^*(E_i - C_i), Z' > + \iiint f_n \delta(\Pi_{i=n-2}^{n})_{\Phi} < \mu_i^*(C_i), Z' >$. Since C_{k_i} is a countable partition of C_i , then C_i can be expressed as a union of disjoint measurable sets $E_i - C_{k_i}$ and C_{k_i} Therefore,

$$\iiint f_n \delta(\Pi_{i=n-2}^{n})_{\Phi} < \mu_i^*(C_i), Z' >= \iiint f_n \delta(\Pi_{i=n-2}^{n})_{\Phi} < \mu_i^*(C_i - C_{k_i}), Z' > + \iiint f_n \delta(\Pi_{i=n-2}^{n})_{\Phi} < \mu_i^*(C_{k_i}), Z' >.$$

By hypothesis, $C_{k_i} \downarrow \varnothing$ .It follows that $\iiint f_n \delta(\Pi_{i=n-2}^{n})_{\Phi} < \mu_i^*(C_{k_i}), Z' >= 0$. (see [1, p. 2]). $\iiint f_n \delta(\Pi_{i=n-2}^{n})_{\Phi} < \mu_i^*(C_i), Z' >= \iiint f_n \delta(\Pi_{i=n-2}^{n})_{\Phi} < \mu_i^*(C_i - C_{k_i}), Z' >$. From the results above, we have

$$\iiint f_n \delta(\Pi_{i=n-2}^{n})_{\Phi} < \mu_i^*(E_i), Z' >= \iiint f_n \delta(\Pi_{i=n-2}^{n})_{\Phi} < \mu_i^*(E_i - C_i), Z' > + \iiint f_n \delta(\Pi_{i=n-2}^{n})_{\Phi} < \mu_i^*(C_i - C_{k_i}), Z' >$$

Proposition 4 Let f be an integrable function. Suppose $\Pi_{i=n-2}^{n}E_{k_i}$ is measurably covered by $\Pi_{i=n-2}^{n}(N(f))_i$ in $\Pi_{i=n-2}^{n}G(R_i)$ such that $\int\int\int f_n\delta\ (\Pi_{i=n-2}^{n})_\Phi < \mu_i^*(N(f)_i), Z' >< M$ for $M>0$. If $\Pi_{i=n-2}^{n}E_{k_i}$ is a countable partition of $\Pi_{i=n-2}^{n}E_i$ in $\Pi_{i=n-2}^{n}R_i$ such that $\Pi_{i=n-2}^{n}\mu_i(E_{k_i}) < M/\varepsilon$, for $\varepsilon>0$, then $\Pi_{i=n-2}^{n}E_i\Theta\Pi_{i=n-2}^{n}(N(f))_i$

Proof. Let $\Pi_{i=n-2}^{n}(N(f))_i = ((a,b,c): f(a,b,c)\neq 0)\in\Pi_{i=n-2}^{n}G(R_i)$. Suppose $\Pi_{i=n-2}^{n}E_{k_i} = ((a,b,c): f_n(a,b,c)>\varepsilon)$ and $\Pi_{i=n-2}^{n}E_i = ((a,b,c): f(a,b,c)>\varepsilon)$ are sets in $\Pi_{i=n-2}^{n}R_i$. Since $\Pi_{i=n-2}^{n}E_{k_i}$ is a countable partition of $\Pi_{i=n-2}^{n}E_i$, we have $E_{k_i}\uparrow E_i$ for $i=n-2,n-1,n$ and each $k\in\aleph$ (see [3 p.20]).So, $\Pi_{i=n-2}^{n}E_i = \cup_{k=1}^{\infty}\Pi_{i=n-2}^{n}E_{k_i}$. Hence, $\varepsilon\chi_{\Pi_{i=n-2}^{n}E_{k_i}} \le \chi_{\Pi_{i=n-2}^{n}E_{k_i}} f_n.$ Let $\mu_{i_{N(f)_i}}^*$ be a vector measure defined on $G(R_i)$ for $i=n-2,n-1,n.$ Since $\Pi_{i=n-2}^{n}E_{k_i}$ is measurably covered by $\Pi_{i=n-2}^{n}(N(f))_i$ for $i=n-2,n-1,n$,we have $E_{K_i}\subset(N(f))_i$. On application of extension procedures, we obtain

$\varepsilon\Pi_{i=n-2}^{n}\mu_{i_{N(f)_i}}^*(E_{k_i}) \le \int\int\int f_n\delta(\Pi_{i=n-2}^{n})_\Phi < \mu_{i_{N(f_i}}^*(E_{k_i}), Z' > \ \Rightarrow\ \varepsilon\Pi_{i=n-2}^{n}\mu_i^*$

$(N(f)_i\cap E_{k_i}) \le \int\int\int f_n\delta(\Pi_{i=n-2}^{n})_\Phi < \mu_i^*(N(f)_i\cap E_{k_i}), Z' > \varepsilon\Pi_{i=n-2}^{n}\mu_i^*(E_{k_i}) \le \int\int\int f_n\delta(\Pi_{i=n-2}^{n})_\phi < \mu_i^*(E_{k_i}), Z' >.$

By hypothesis, $\int\int\int f_n\delta(\Pi_{i=n-2}^{n})_\Phi < \mu_i^*(N(f)_i), Z' >< M$ and $\Pi_{i=n-2}^{n}E_{k_i}\ \ \Theta\Pi_{i=n-2}^{n}N(f)_i.$ Therefore,

$\varepsilon\chi_{\Pi_{i=n-2}^{n}\mu_i^*(E_{k_i}}) \le \int\int\int f_n\delta(\Pi_{i=n-2}^{n})_\Phi < \mu_i^*(E_{k_i}), Z' >< M \Rightarrow \Pi_{i=n-2}^{n}\mu_i^*(E_{k_i}) \le M/\varepsilon.$ Since

$\Pi_{i=n-2}^{n}E_i = \cup_{k=1}^{\infty}\Pi_{i=n-2}^{n}E_{k_i}$, we have $\Pi_{i=n-2}^{n}\mu_i^*(E_i) \le M/\varepsilon \Rightarrow \Pi_{i=n-2}^{n}E_i\Theta\,\Pi_{i=n-2}^{n}(N(f))_i.$

Proposition 5 Let $\Pi_{i=n-2}^{n}E_i$ be measurably covered by $\Pi_{i=n-2}^{n}F_i$. Suppose $\Pi_{i=n-2}^{n}G_i$ is a measurable set such that $\Pi_{i=n-2}^{n}G_i\subset\Pi_{i=n-2}^{n}(F_i-E_i)$. If $(a_i,b_i,c_i): f_n(a_i,b_i,c_i)\ge\varepsilon)$ is a finite partition of G_i for each $i=n-2,n-1,n$ and $n\in\aleph$ such that $f_n\downarrow 0$, then $\Pi_{i=n-2}^{n}((a_i,b_i,c_i): f_n(a_i,b_i,c_i)\ge\varepsilon)$ is an empty set.

Proof. Let $\Pi_{i=n-2}^{n}G_i = N(f_1), \Pi_{i=n-2}^{n}E_i = \Pi_{i=n-2}^{n}((a_i,b_i,c_i): f_n(a_i,b_i,c_i)\ge\varepsilon)$ and $M = max(f_1(a_i,b_i,c_i)) \Rightarrow \chi_{\Pi_{i=n-2}^{n}G_i} f_1 \le f_1.$ Since $f_n\downarrow 0$, it follows that $f_n\le M\chi_{\Pi_{i=n-2}^{n}G_i}$ for all n. So $N(f_n)\subset N(f_1) = \Pi_{i=n-2}^{n}G_i \Rightarrow \chi_{\Pi_{i=n-2}^{n}G_i} f_n = f_n$. (see [2 , p. 5]) Since $\Pi_{i=n-2}^{n}E_i$ is a finite partition of $\Pi_{i=n-2}^{n}G_i$, it follows that $\Pi_{i=n-2}^{n}G_i = \Pi_{i=n-2}^{n}((G_i-E_i)\cup E_i)$.Following disjoint partition of $\Pi_{i=n-2}^{n}G_i$ (see [4, p. 6]) and on application of measurable cover estimate technique, we obtain

$\int\int\int f_n\delta(\Pi_{i=n-2}^{n})_\Phi < \mu_i^*(G_i), Z' >= \int\int\int f_n\delta(\Pi_{i=n-2}^{n})_\Phi < \mu_i^*(G_i-E_i), Z' > + \int\int\int f_n\delta(\Pi_{i=n-2}^{n})_\Phi < \mu_i^*(E_i), Z' >.$

Since $f_n(a_i,b_i,c_i)<\varepsilon$ on $\Pi_{i=n-2}^{n}(G_i-E_i)$,it follows that $\int\int\int f_n\delta(\Pi_{i=n-2}^{n})_\Phi < \mu_i^*(G_i-E_i), Z' > \le\varepsilon$ $\Pi_{i=n-2}^{n}\mu_i^*(G_i-E_i) \le \varepsilon\Pi_{i=n-2}^{n}\mu_i^*(G_i).$ Replacing $\Pi_{i=n-2}^{n}G_i$ with $\Pi_{i=1}^{n}E_i$ in the inequality $f_n\le M\chi_{\Pi_{i=n-2}^{n}G_i}$, we obtain $\chi_{\Pi_{i=n-2}^{n}E_i} f_n \le M\chi_{\Pi_{i=n-2}^{n}E_i}$. Therefore,

$\int\int\int f_n\delta(\Pi_{i=n-2}^{n})_\Phi < \mu_i^*(G_i), Z' >\le \varepsilon\Pi_{i=n-2}^{n}\mu_i^*(G_i) + M\Pi_{i=n-2}^{n}\mu_i^*(E_i).$ Since $\Pi_{i=n-2}^{n}G_i\subset\Pi_{i=n-2}^{n}(F_i-E_i)$ and $\Pi_{i=n-2}^{n}E_i\Theta\Pi_{i=n-2}^{n}F_i$ (by hypothesis), it follows that $\Pi_{i=n-2}^{n}\mu_i^*(G_i)=0$. Since $f_n\downarrow$ 0 for each n, it follows that $\int\int\int f_n\delta(\Pi_{i=n-2}^{n})_\Phi < \mu_i^*(G_i), Z' >= 0.$ Therefore, $\Pi_{i=n-2}^{n}E_i = \Pi_{i=n-2}^{n}((a_i,b_i,c_i): f_n(a_i,b_i,c_i)\ge\varepsilon) = \varnothing.$

Theorem 1 Let f_n be an integrable function with respect to $(\Pi_{i=n-2}^{n})_\Phi < \mu_i^*, Z' >$. For each $\varepsilon>0$, the set $\Pi_{i=n-2}^{n}A_i$ is measurably covered by $\Pi_{i=n-2}^{n}E_i = N(f_n)$ if $\int\int\int f_n\delta(\Pi_{i=n-2}^{n})_\Phi < \mu_i^*(E_i-A_i), Z' >< \varepsilon$.

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Proof. Let $\Pi_{i=n-2}^{n} E_i = ((a,b,c): f_n(a,b,c) \neq 0)$. Then, $\chi_{\Pi_{i=n-2}^{n} E_i} f_n \leq f_n$ where $\mu_i^*(E_i) < \infty$ for $i = n-2, n-1, n.$ Let $P_o = \Pi_{i=n-2}^{n} A_{k_i}$ be a countable partition of $\Pi_{i=n-2}^{n} A_i$ such that for every countable partition $P = \Pi_{i=n-2}^{n} E_{k_i}$ of $\Pi_{i=n-2}^{n} E_i$ in $\Pi_{i=n-2}^{n} G(R_i)$ we have $P_o \Theta P.$ Since $\Pi_{i=n-2}^{n} A_i = \cup_{k=1}^{\infty} \Pi_{i=n-2}^{n} A_{k_i}$ and $\Pi_{i=n-2}^{n} E_i = \cup_{k=1}^{\infty} \Pi_{i=n-2}^{n} E_{k_i}$, applying measurable cover estimate technique, we obtain $\Sigma_{k=1}^{\infty} \iiint f_n \delta(\Pi_{i=n-2}^{n})_{\Phi} < \mu_i^*(E_{k_i} - A_{k_i}), Z' >< \varepsilon/3.$ Consider the the partition $P_{m'} = (\Pi_{i=n-2}^{n} C_{k_i} : k = 1,2,.....)$ of $P - P_o = \Pi_{i=n-2}^{n}(E_{k_i} - A_{k_i})$ such that $\Pi_{i=n-2}^{n}(E_{k_i} - A_{k_i}) = \cup_{k=1}^{\infty} \Pi_{i=n-2}^{n} C_{k_i}$ and $(P - P_o)\Theta P_{m'} \iiint f_n \delta(\Pi_{i=n-2}^{n})_{\Phi} < \mu_i^*(\cup_{k=1}^{\infty} C_{k_i} - (E_{k_i} - A_{k_i})), Z' >< \varepsilon/3.$ There is a sequence $(P_{k_i}')_{k \in \aleph} = \Pi_{i=n-2}^{n} B_{k_i}$ of a countable partitions of $P_{m'}$ in $\Pi_{i=1}^{3} G(R_i)$ such that $\Pi_{i=n-2}^{n} C_{k_i} = \cup_{k=1}^{\infty} \Pi_{i=n-2}^{n} B_{k_i}$ and $P_{m'} \Theta P_{k_i}' \Rightarrow \iiint f_n \delta(\Pi_{i=n-2}^{n})_{\Phi} < \mu_i^*(\cup_{k=1}^{\infty} B_{k_i} - C_{k_i}), Z' >< \varepsilon/3.$ Therefore, $\iiint f_n \delta(\Pi_{i=n-2}^{n})^* < \mu_i^*(E_i - A_i), Z' > \leq \Sigma_{k=1}^{\infty} \iiint f_n \delta(\Pi_{i=n-2}^{n})^* < \mu_i^*(E_{k_i} - A_{k_i}), Z' > + \iiint f_n \delta(\Pi_{i=n-2}^{n})_{\Phi} < \mu_i^*(\cup_{k=1}^{\infty} C_{k_i} - (E_{k_i} - A_{k_i})), Z' > + \iiint f_n \delta(\Pi_{i=n-2}^{n})_{\Phi} < \mu_i^*(\cup_{k=1}^{\infty} B_{k_i} - C_{k_i}), Z' >< \varepsilon/3 + \varepsilon/3 + \varepsilon/3 = \varepsilon \Rightarrow \iiint f_n \delta(\Pi_{i=n-2}^{n})_{\Phi} < \mu_i^*(E_i - A_i), Z' >< \varepsilon.$

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