



*(Knowledge for Development)*

**KIBABII UNIVERSITY**  
**UNIVERSITY EXAMINATIONS**  
**2021/2022 ACADEMIC YEAR**  
**FIRST YEAR SECOND SEMESTER**  
**SUPPLEMENTARY/SPECIAL EXAMINATION**  
**FOR THE DEGREE OF BACHELOR OF EDUCATION AND**  
**BACHELOR OF SCIENCE IN COMPUTER SCIENCE**

**COURSE CODE:** MAP 122/MAT 212

**COURSE TITLE:** LINEAR ALGEBRA I

**DATE:** 28/07/2022      **TIME:** 8:00 AM -10 AM

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**INSTRUCTIONS TO CANDIDATES**

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

**QUESTION ONE (30 MARKS)**

a) Define the following terms:

- i. Trace of matrix (1 mk)
- ii. Linear combination of a vector (2 mks)
- iii. Transpose of a matrix (1 mk)
- iv. Vector space (2 mks)

b) Let A and B be invertible matrices. Prove that  $(AB)^{-1}=B^{-1}A^{-1}$  (3 marks)

c) Let  $AX=B$  be system of linear equation. Show that if  $A^{-1}$  exists, the solution is unique and is given by  $X=A^{-1}B$  (3marks)

d) Prove that the following transformation  $hT: R^2 \rightarrow R^2$  is linear.  $T(x, y) = (2x, x + y)$  ( mks)

e) Find AB given that

$$A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}, B = A^T \quad (3 \text{ mks})$$

f) Use the row reduction formula to find the inverse of the matrix

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 2 & 1 & 6 \end{bmatrix} \quad (8 \text{ mks})$$

g) Determine the basis of the matrix B below;

$$B = \begin{pmatrix} 1 & -3 & 2 \\ -2 & 6 & -4 \\ -1 & 3 & -2 \end{pmatrix} \quad (3\text{marks})$$

**QUESTION TWO (20 MARKS)**

a) Given the matrix

$$A = \begin{bmatrix} 1 & -1 & 2 \\ -3 & 1 & 2 \\ 3 & -2 & -1 \end{bmatrix}$$

Determine:

- i) The determinant of A
- ii) The matrix of the minors
- iii) The adjoint of the co-factors of A
- iv) Inverse of A

(12mks)

b) Determine whether the function  $f(x)=x^2+4x+5$  is a linear combination of the functions  $g(x)=x^2+x-1$  and  $h(x)=x^2+2x+1$  (5marks)

c) Determine the rank of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 2 & 5 & 8 \end{bmatrix}$$

(3mks)

**QUESTION THREE (20MARKS)**

a) Use the Cramer's rule to solve the following system of linear equations.(10 mks)

$$x + y + z = 6$$

$$2x + y = 4$$

$$2x + 3y + z = 11$$

b) Use Gaussian elimination to solve the system of equations (6 mks)

$$2x - y + z = 1$$

$$2x + 2y + 2z = 2$$

$$-2x + 4y + z = 5$$

c) Determine whether the set defined by the vector  $(a, b, 2a + 3b)$

Is a subspace of  $\mathbb{R}^3$

(4 mks)

**QUESTION FOUR (20MARKS)**

a) If  $A = \begin{pmatrix} 1 & 2 & 3 & 2 \\ 5 & 6 & 7 & 2 \\ 8 & 9 & 10 & 7 \end{pmatrix}$ ,  $B = \begin{pmatrix} 1 & 2 \\ 5 & 6 \\ 8 & 9 \\ 3 & 7 \end{pmatrix}$ , Find AB (5 mks)

b) Find the determinant of matrix below by reducing it first to an upper triangular

matrix .  $A = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ 2 & 1 & -1 \end{pmatrix}$

(5mks)

c) State (with brief explanation) whether the following statement is true or false. The vectors  $(1, 0, 0)$ ,

$$(0, 2, 0), (1, 2, 0) \text{ span } \mathbb{R}^3$$

(5 mks)

d) Determine whether the vectors  $(1, 2, 0)$ ,  $(0, 1, -1)$ ,  $(1, 1, 2)$  are linearly independent in  $\mathbb{R}^3$

(5 mks)

**QUESTION FIVE(20MARKS)**

a) Express  $V = (1, -2, 5)$  in  $\mathbb{R}^3$  as a linear combination of the vectors  $u_1 = (1, 1, 1)$ ,  $u_2 = (1, 2, 3)$  and

$$u_3 = (2, -1, 1)$$

(6 mks)

b) i) Define the basis of a vector space. (2 mks)

ii) Prove that the vectors  $(1, 1, 1)$ ,  $(0, 1, 2)$  and  $(3, 0, 1)$  form a basis for  $\mathbb{R}^3$ (6 mks)

c) i) Define linear transformation. (2 mks)

ii) Verify for the transformation defined by the matrix  $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$  that

$$A(V_1 + V_2) = AV_1 + AV_2$$

(4 mks)