(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER

SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF INFORMATION TECHNOLOGY

COURSE CODE: **MAT 121**

COURSE TITLE: **LINEAR ALGEBRA I**

DATE: 30/9/2021 **TIME:** **2 PM - 4 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE COMPULSORY (30 MARKS)

a) Let $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 3 & 0 \\ 1 & 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ 2 & 0 \\ 1 & -2 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ 0 & 3 \end{bmatrix}$

i. Find A^{-1}. (3marks)

ii. Find a matrix X such that $AX + B = C$. (4marks)

iii. Is it possible to find a matrix Y such that $YA + B = C$? Explain. (4marks)

b) Let $A = \begin{bmatrix} 7 & 3 \\ -2 & -5 \end{bmatrix}$ and $X = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$. Compute, if possible, $X^T A^T$, $A^T X^T$, $X^T X$, and XX^T

(10marks)

c) Let $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$. Find C if $AC = B^T$. (5marks)

d) Find a vector X, of length 6, in the opposite direction of $Y = (1, 2, -2)$. (4marks)

QUESTION TWO (20 MARKS)

a) Solve the system (6marks)

$$\begin{aligned} x + y + z &= 0 \\ x + 2y + 3z &= 0 \\ x + 3y + 4z &= 0 \\ x + 4y + 5z &= 0. \end{aligned}$$

b) Show that if $A^{-1} = A^T$, then $|A| = 1 \text{ or } |A| = -1$ (3marks)

c) If A and B are 2×2 matrices with det $(A) = 2$ and det $(B) = 5$, compute $|3A^2(AB^{-1})^T|$ (5marks)

d) Let $x, y \in \mathbb{R}^n$ such that $\| x \| = 2$ and $\| y \| = 3$ and the angle between them is $\frac{\Pi}{3}$. Evaluate $\| x - y \|$

(6marks)

QUESTION THREE (20 MARKS)

a) Find the reduced row echelon form (r.r.e.f.) of the following matrix: (5marks)

$$\begin{bmatrix} 2 & 4 & 6 & 0 \\ 1 & 2 & 4 & 0 \\ 1 & 3 & 3 & 1 \end{bmatrix}$$

b) Let A be a non-singular 4×4 matrix with $|A^{-1}| = 3$. Find

i. $|\text{adj}(A)|$ (4marks)

ii. $|\frac{1}{2} A^T Adj(A^{-1})|$ (6marks)

c) Find all vectors in $\mathbb{R}^4$ which are perpendicular to the vectors $X = (1, 1, 2, 2)$ and $Y = (2, 3, 5, 5)$.

(5marks)

QUESTION FOUR (20 MARKS)

a) Let **u** and **v** be two solutions of the non-homogenous system $A\mathbf{x} = B$. Show that $\mathbf{u} - \mathbf{v}$ is a solution to the homogenous system $A\mathbf{x} = O$. (4marks)

b) Verify that the triangle with vertices $A(1, 1, 2)$, $B(1, 2, 3)$, and $C(3, 0, 3)$ is a right triangle. (5marks)

c) Solve the following linear system (6marks)

$$x + 3y - z + w = 1$$
$$2x - y - 2z + 2w = 2$$
$$3x + y - z + w = 1$$

d) Show that if **x** and **y** are in $\mathbb{R}^n$, then $\| x + y \| \leq \| x \| + \| y \|$ (5marks)

QUESTION FIVE (20 MARKS)

a) Let **u** and **v** be two solutions of the homogenous system $A\mathbf{x} = O$. Show that $r\mathbf{u} + s\mathbf{v}$ (for $r, s \in \mathbb{R}$) is a solution to the same system. (4marks)

b) Show that $U \cdot (V + W) = U \cdot V + U \cdot W$, for any vectors $U, V, W \in \mathbb{R}^n$. (4marks)

c) Let A be a square matrix. Show that if $A = 2A^T$, then $A = O$. (4marks)

d) Find all values of a for which $X = (a^2 - a, -3, -1)$ and $Y = (2, a - 1, 2a)$ are orthogonal. (5marks)

e) Show that if C_1 and C_2 are solutions of the system $A\mathbf{x} = B$, then $4C_1 - 3C_2$ is also a solution of this system. (3marks)