(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF MASTER OF SCIENCE IN STATISTICS

COURSE CODE: **STA 805**

COURSE TITLE: MULTIVARIATE ANALYSIS

DATE: 07/10/2022 **TIME:** 2 PM -4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other two

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

a) Explain the following terms (2 mks)

i. Multivariate analysis

ii. Mahalanobis distance

b) State and briefly discuss any three applications of multivariate analysis in a real life situation (3 mks)

c) Given

$$\sum = \begin{pmatrix} 4 & 1 & 1 \\ 1 & 9 & 1 \\ 1 & 1 & 16 \end{pmatrix}$$

Find the correlation Matrix ρ (5 mks)

d) Given $f(x_1, x_2, x_3) = \begin{cases} 8x_1x_2x_3 & 0 < x_i < 1,\ i = 1,2,3, \ldots. \\ 0 & elsewhere \end{cases}$

Determine whether $x_1, x_2\ and\ x_3$ are Independent (10 mks)

e) Given $X = [1 \quad 2 \quad 3]$ and $\Sigma = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 4 \end{pmatrix}$ where

$X \sim N_3(\mu, \Sigma)$

and suppose $Y = X_1 - 2X_2 + X_3$. Compute the Mean and Covariance of Y

(10 mks)

QUESTION TWO (20 MARKS)

Assume that we have a random samples $X_1, X_2, \ldots \ldots \ldots X_n$ all having the same distribution with mean μ and variance – covariance matrix Σ. Show that

a) $\bar{X}$ *is unbiased estimator of* μ *and has variance – covariance matrix* $\frac{1}{n}\Sigma$

b) $\frac{n}{n-1} S_n$ is unbiased estimator of Σ (8 mks)

c) S_n is a biased estimator of Σ and bias is $= \frac{-1}{n}\Sigma$ (4 mks)

QUESTION THREE (20 MARKS)

a) By use of an example state the importance of partial correlation in multivariate analysis and the significance of controlling some variables. (5 mks)

b) Given $X \sim N_4$

$$\left(\begin{bmatrix} 1 \\ 2 \\ 0 \\ 3 \end{bmatrix} \quad \begin{bmatrix} 4 & 0 & 1 & 3 \\ 0 & 4 & 1 & 1 \\ 1 & 1 & 3 & 1 \\ 3 & 1 & 1 & 9 \end{bmatrix} \right)$$

Find the conditional distribution of $\binom{X_1}{X_2}$ given $X_2 = x_2$ and $X_4 = x_4$, hence the partial correlation between X_1 and X_3 holding the values of X_2 and X_4 constant.

(15 mks)

QUESTION FOUR (20 MARKS)

If $X_1, X_2, \dots \dots \dots X_n$ is a random sample from a population with mean vector $\underline{\mu}$ and variance – covariance matrix Σ, where

$$\underline{\mu} = \begin{bmatrix} \mu_1 \\ \dots \\ \mu_p \end{bmatrix}$$

Show that

$\bar{X}$ Is a consistent estimator for μ and S is consistent for Σ for any $\varepsilon > 0$ i.e.

$$Pr\left(\left\| \bar{X} - \mu \right\| \varepsilon > 0 \right) \longrightarrow 0 \text{ as } n \to \infty \text{ and } Pr\left(\left\| S - \Sigma \right\| \varepsilon > 0 \right) \longrightarrow 0 \text{ as } n \to \infty$$

QUESTION FIVE (20 MARKS)

Derive the maximum likelihood estimator of μ and Σ from a multivariate normal distribution (MNV) $X \sim N_p(\mu \quad \Sigma)$