(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF MASTER OF SCIENCE IN STATISTICS

COURSE CODE: **STA 802**

COURSE TITLE: TEST OF HYPOTHESIS

DATE: 06/10/2022 **TIME:** 2 PM -4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other two

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) Explain the following terms (10 mks)

i. Statistical hypothesis

ii. Sequential test

iii. Simple hypothesis and Composite hypothesis

iv. Type I Error and Type II Error

v. Critical region

vi. Power function

vii. Significance level

viii. One-parameter exponential family

b) State Neyman Pearson Lemma and prove it (10 mks)

c) Let $X_1, X_2, \dots\dots\dots X_n$ be a random sample from a normal distribution $N(\mu, \delta^2)$ where δ^2 is known. Test the hypothesis that

$H_0: \mu = \mu_0$ Vs $H_1: \mu > \mu_0$ (10 mks)

QUESTION TWO (20 MARKS)

a) Suppose X has Bernoulli distribution with probability of success θ. On the basis of a random sample of size n, obtain the test for $H_0: \theta \leq \theta_0$ Vs $H_1: \theta > \theta_0$ (10 mks)

b) Let $X_1, X_2, \dots\dots\dots X_n$ be a random sample from $N(\mu, 1)$. Test the hypothesis $H_0: \mu \leq \mu_0$ Vs $H_1: \mu = \mu_1$. Take the least favourable distribution as

$$\beta(\mu) = \begin{cases} 1 \ if \ \mu = \mu_0 \\ 0 \ otherwise \end{cases}$$ (10 mks)

QUESTION THREE (20 MARKS)

a) What is a UMP test? (1 mk)

b) Show that the power of the test is always greater than its size i.e. $1 - \beta \geq \alpha$ (5 mks)

c) What do you understand by

i. Randomized test (1 mk)

ii. Families of densities $p(x;\theta) : \theta \in \Omega$ have a likelihood ratio. (3 mks)

d) Let X be a random variable distributed as normal law with mean μ and variance unity. Using monotomic likelihood ratio principle, obtain the UMPT for

$H_0 : \mu < \mu_0 vs H_1 : \mu > \mu_0$ (10 mks)

QUESTION FOUR (20 MARKS)

a) Obtain the region for testing $\theta = \theta_0 \ vs \theta_1 > \theta_0 \ and \ \theta \leq \theta_1 < \theta_0$ in the case of normal population $N(\theta, \sigma^2)$, where σ^2 is known. Hence find the power of the test. (10 mks)

b) Show that for the normal distribution with mean 0 and σ^2 variance, the best critical region for $H_0 : \sigma = \sigma_0 vs H_1 : \sigma = \sigma_1$ is of the form

$$\sum_{i=0}^{n} X_i^2 \leq a_\alpha \ for \ \sigma_0 > \sigma_1 \ and \ \sum_{i=0}^{n} X_i^2 \geq b_\alpha \ for \ \sigma_0 < \sigma_1$$

Obtain the power of the critical region when $\sigma_0 > \sigma$ (10 mks)

QUESTION FIVE (20 MARKS)

a) Obtain the statistics for testing the hypothesis that the mean of a Poisson population is 2 against the alternative that its 3 on the basis of n independent observation. (5 mks)

b) Suppose you want to test $H_0 : \lambda = 2 vs H_1 : \lambda = 1 \ where \ \lambda$ is the parameter of the Poisson distribution. Obtain the best critical region of the test. (5 mks)

c) Suppose a random sample of size n is taken from the Poisson population. Give the MPT of size α for testing the hypothesis

$H_0 : \lambda = \lambda_0 vs H_1 : \lambda = \lambda_1 \ (\lambda_1 > \lambda_0)$

How can you use the above results for finding the confidence interval for λ? Obtain the power function of the test of the hypothesis

$H_0 : \lambda = \lambda_0 vs H_1 : \lambda > \lambda_0$ (10 mks)