(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

FORTH YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: **MAP 422**

COURSE TITLE: **MEASURE THEORY AND INTEGRATION**

DATE: 09/09/2022 **TIME:** 2:00 PM - 4:00 AM

Answer question ONE and any other TWO Questions

TIME: 2 Hours

QUESTION ONE (20 MARKS)

a) Define the following terms

   i. Union
   ii. Intersection
   iii. Difference
   iv. Symmetric difference
   v. Family of elements

b) Given R is a ring of subsets of a set X, show that the monotone ring generated by R coincides with the ring generated by R; $M(R) = G(R)$

c) State the Lemma on Monotone classes (LCM)

QUESTION TWO (20 MARKS)

a) Define the following terms

   i. Set function
   ii. Measure
   iii. Finite set function
   iv. Positive set function

b) Given μ is a measure on a ring R, show that

   i. μ is monotonic, that is $\mu(E) \leq \mu(F)$ whenever E and F are sets in R such that $E \subset F$
   ii. μ is conditionally subtractive that is $\mu(E - F) \leq \mu(F) - \mu(E)$ whenever E and F are sets in R such that $E \subset F$ and $\mu(E)$ is finite
   iii. μ is finitely additive that is if $E_1, ..., E_n$ are mutually disjoint sets in R, then $\mu(\cup_1^n E_n) = \sum_1^n \mu(E_n)$
   iv. μ is countably additive that is, if E_k is a sequence of mutually disjoint sets in R such that $\cup_1^\infty E_k$ is in R, then $\mu(\cup_1^\infty E_k) = \sum_1^\infty \mu(E_k)$ in the sense that the LUB of the (increasing) sequence of partial sums $\sum_1^\infty \mu(E_k)$ is equal to $\mu(\cup_1^\infty E_k)$

QUESTION THREE (20 MARKS)

a) Show that if v is an outer measure, the class M of v-measurable sets is a ring

b) State the unique Extension Theorem (UET)

c) Show that if E is a set in R, and $E = \cup_1^r [a_i b_i) = \cup_1^s [c_j d_j)$ are two representations of E, then $\sum_1^r (b_i - a_i) = \sum_1^s (d_j - c_j)$