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Suborbital Graphs and their Properties for ordered Triples in An (n = 5, 6, 7) Through Rank and Subdegree Determination

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Abstract

In this paper, through computation of the rank and subdegrees of alternating group A_n (n=5,6,7) on ordered triples we construct the suborbital graphs corresponding to the suborbits of these triples. When $A_n(n \geq 5)$ acts on ordered pairs the suborbital graphs corresponding to the non-trivial suborbits are found to be disconnected. Further, we investigate properties of the suborbital graphs constructed.

Keywords: Rank, subdegrees, ordered triple of an alternating group, suborbital graphs.

Mathematics Subject Classification: Primary 05E18; Secondary 05E30, 14N10, 05E15.

Introduction

In 1967, Sims[6] introduced suborbital graphs corresponding to the non-trivial suborbits of a group. He called them orbitals. In1977, Neumann [4] extended the work of Higman [2] and Sims [6] to finite permutation groups, edge coloured graphs and Matrices. He constructed the famous Peterson graph as a suborbital graph corresponding to one of the nontrivial suborbits of S_5 acting on unordered pairs from the set X={1,2,3,4,5}. The Peterson graph was first introduced by Petersen in 1898 [5]. In1992, Kamuti[3] devised a method for constructing some of the suborbital graphs of PSL (2,q) and PGL (2,q) acting on the cosets of their Maximal dihedral sub-groups of orders $q-1$ and $2(q-1)$ respectively. This method gave an alternative way of constructing the Coxeter graph which was first constructed by Coxeter in 1986[1]. In this paper, through computation of the rank and subdegrees of alternating group A_n (n=5,6,7) on unordered triples, we construct the suborbital graphs corresponding to the suborbits of these triples and further investigate properties of the suborbital graphs constructed.

Preliminaries

Notation and Terminology

We first present some basic notions and terminologies as used in the context of graphs and suborbital graphs that shall be used in the sequel

A_n -Alternating group of degree n and order $\frac{n!}{2}$; $|G|$-The order of a group G; $X^{[3]}$ -The set of an ordered triples from set X= {1,2,…,n}; [a,b,c] - Ordered triple;

Definition 1

A graph G is an ordered pair (V,E), where V is a non-empty finite set of vertices and E is a set of pairs of distinct vertices in G, called edges. A loop is an edge from a vertex to itself.

Definition 2

A multigraph is a graph which is allowed to have multiple edges, but no loops.

Definition 3

If e = {u, v} is an edge of a graph G, then u and v are the end vertices of e, and we say u and v are adjacent in G. This relation is often denoted by u~v.

Definition 4

The degree or valency dG(v) of a vertex v of a graph G is the number of edges incident to v. A vertex of degree O is an isolated vertex.

Definition 5

A walk of length k joining u and v in G is a sequence of vertices and edges of G of the form v_0, e_1, v_1, e_2, v_2, ---, v_{k-1}, e_k, v_k, where v_0=u,v_k=v and e_i={v_{i-1},v_i} for i=1,2,---,k. A walk joining u and v is closed if u=v, and is a path if no two vertices of the walk (except possibly u and v) are equal; a closed path is called a circuit. Note that the edges e_1,---,e_k will frequently be omitted from the definition of a walk.

Definition 6

A graph G is connected if every pair of vertices of G is joined by some path; otherwise, G is disconnected.

Definition 7

A graph D, or a directed graph consists of a finite non-empty set V=D(V) of vertices together with a collection of ordered pairs of distinct vertices of V.

Definition 8

Let G be transitive on X and let Gx be the stabilizer of a point $x \in X$. The orbits Δ0= {x}, Δ1, Δ2,---, Δr-1 of Gx on X are called the suborbits of G. The rank of G is r and the sizes ni = |Δi| (i=0,1,---,r-i), often called the 'lengths' of the suborbits, are known as the subdegrees of G. Note that both r and the cardinalities of the suborbits Δi (i=0, 1, ---, r-1) are independent of the choice of $x \in X$.

Definition 9

Let Δ be an orbit of Gx on X. Define Δ*= {gx| $g \in G$, $x \in g\Delta$ }, then Δ* is also an orbit of Gx and is called the Gx-orbit (or the G-suborbit) paired with Δ. Clearly |Δ| = |Δ*|. If Δ*= Δ, then Δ is called a self-paired orbit of Gx.

Theorem 10 [Wielandt 1964]

Gx has an orbit different from {x} and paired with itself if and only if G has even order. Observe that G acts on X x X by g(x, y)=(gx, gy), $g \in G$, $x, y \in X$.

If O $\subseteq$ X x X is a G-orbit, then for a fixed x ∈ X, Δ= {y ∈ X | (x, y) ∈ O} is a G_x-orbit.

Conversely if Δ $\subseteq$ X is a G_x-orbit, then O = {(gx, gy) | g ∈ G, y ∈ Δ} is a G-orbit on X x X.

We say that Δ corresponds to O. The G- orbits on X x X are called suborbitals. Let $O_i \subseteq$ X x X, i = 0, 1,--

-,r-1 be a suborbital. Then we form a suborbital graph Γ_i, by taking X as the set of vertices of Γ_i and by including a directed edge from x to y ($x,y \in X$) if and only if $(x, y) \in O_i$. Thus each suborbital O_i determines a suborbital graph Γ_i . Now $O_i^*= \{(x, y) \mid (y, x) \in O_i\}$ is a G-orbit.

Theorem 11 [Sims 1967]

Let Γ_i^* be the suborbital graph corresponding to the suborbital O_i^*. Let the suborbit Δ_i (i=0,1,---,r-1) correspond to the suborbital O_i. Then Γ_i is undirected if Δ_i is self-paired and Γ_i is directed if Δ_i is not self-paired.

Theorem 12[Sims 1967]

Let G be transitive on X. Then G is primitive if and only if each suborbital graph Γi (i=1,2,---,r-1) is connected.

Theorem 13 [Wielandt 1964]

Let G be transitive on X and let Gx be the stabilizer of the point x ∈ X. Let Δ0 = {x}, Δ1, Δ2,---, Δk-1 be orbits of Gx on X of lengths no=1, n1,n2,---,nk-1, where no≤ n1≤ n2≤ ---≤nk-1. If there exists an index j>0 such that nj>n1nj-1, then G is imprimitive on X.

Suborbital Graphs of $G=A_n$ Acting on $X^{[3]}$

In this section we construct and discuss the properties of the suborbital graphs of $G=A_n$ acting on $X^{[3]}$.

Suborbital Graphs of $G=A_5$ Acting on $X^{[3]}$ and their properties

The suborbits Δ_0, Δ_1, ---, Δ_{59} of G are given in Appendix I. By Definition 9, the suborbits Δ_i for which i=(1, 2, 4, 7, 9, 10, 11, 12, 14, 16, 25, 27, 29, 42, 43, 44, 47, 54, 55, 56 and 59) are self-paired. Therefore, the corresponding suborbital graphs are undirected by Theorem 11.

On the other hand, the suborbits Δ_j for which j = (3, 5, 13, 15, 17, 18, 19, 20, 21, 30, 31, 32, 33, 40, 41, 45, 52, 53 and 57) are paired with the suborbits Δ_k, k= (6, 8, 24, 26, 28, 36, 38, 48, 50, 37, 39, 49, 51, 22, 35, 46, 23, 34 and 58) respectively. Thus for each j and k, the corresponding suborbital graphs are directed by Theorem 11.

Suborbital Graphs of $G=A_6$ Acting on $X^{[3]}$ and their properties

The suborbits Δ_0, Δ_1, ---, Δ_{43} of G are given in Appendix II. By Definition 9, the suborbits Δ_i for which i=(1, 2, 5, 6, 7, 14, 16, 21, 22, 24, 25, 32, 33, 40, 41, 42 and 43) are self-paired. Therefore, the corresponding suborbital graphs are undirected.

On the other hand , the suborbits Δ_j for which j= (3, 29, 30, 31, 37, 38, 9, 10, 8, 11, 15, 17 and 35) are paired with the suborbits Δ_k, k= (4, 36, 26, 27, 28, 34, 12, 18, 19, 13, 20, 23 and 39) respectively. Thus for each j and k, the corresponding suborbital graphs are directed by Theorem 11.

Theorem 14 $G=A_6$ acts imprimitively on $X^{[3]}$.

Proof Consider the orbits Δ_0 = [1, 2, 3], Δ_1, Δ_2, ---, Δ_{43} of $G_{[1, 2, 3]}$ on $X^{[3]}$. Suppose the lengths of these orbits are n_0, n_1, n_2, ---, n_{43}, where $n_0 \le n_1 \le n_2 \le --- \le n_{43}$. Then from Table 15, below,

Table 14: Subdegrees of A_6 on $X^{[3]}$

Suborbit length	1	2	3
Number of suborbits	6	18	20

$n_1 = 1$, $n_5 = 1$ and $n_6 = 2$. Now let j = 6, then j > 0 and $n_j = 2 > (1)(1) = n_1 n_{j-1}$. Hence by Theorem 13, A_6 acts imprimitively on $X^{[3]}$. By Theorem 12, all the corresponding suborbital graphs are disconnected.

Suborbital Graphs of $G=A_7$ Acting on $X^{[3]}$ and their properties

The suborbits Δ_0, Δ_1, ---, Δ_{34} of G are given in Appendix III. By Definition 9, the suborbits Δ_i for which i= (1, 2, 5, 6, 7, 14, 15, 22, 23, 24, 28, 32, 33 and 34) are self-paired. Therefore, the corresponding suborbital graphs are undirected. On the other hand, the suborbits Δ_j for which j= (3, 8, 9, 10, 11, 16, 17, 25, 26 and 29) are paired with the suborbits Δ_k, k= (4, 12, 13, 19, 18, 20, 21, 27, 30 and 31) respectively. Thus for each j and k, the corresponding suborbital graphs are directed by Theorem 11.

Theorem 15 G=A7 acts imprimitively on X[3].

Proof Consider the orbits Δ_0 = [1, 2, 3], Δ_1, Δ_2, ---, Δ_{34} of $G_{[1,2,3]}$ on $X^{[3]}$. Suppose the lengths of these orbits are n_0, n_1, n_2, ---, n_{34}, where $n_0 \le n_1 \le n_2 \le --- \le n_{34}$. Then from Table 3.3.2, below,

Table 3.3.2: Subdegrees of A_7 on $X^{[3]}$

Suborbit length	1	4	12
Number of suborbits	6	18	11

$n_1 = 1$, $n_5 = 1$ and $n_6 = 4$. Now let j = 6, then j > 0 and $n_j = 4 > (1)(1) = n_1 n_{j-1}$. Hence by Theorem 13, A_7 acts imprimitively on $X^{[3]}$. By Theorem 12, all the corresponding suborbital graphs are disconnected.

References

1. Coxeter, H. S. M. (1986). My graph, Proceedings of London Mathematical Society 46: 117 – 135.
2. Higman, D. G. (1964). Finite permutation groups of rank 3.Math.Zeitschrifl 86: 145 – 156.
3. Kamuti, I. N. (1992). Combinatorial formulas, invariants and structures associated with primitive permutation representations of PSL (2, q) and PGL (2, q). PhD. Thesis, University of Southampton, U.K.
4. Neumann, P. M. (1977). Finite permutation groups Edge – coloured graphs and matrices, edited by M. P. J. Curran, Academic Press, London.
5. Petersen, J. (1898). Sur le. Theore'me de Tait Intermed Math 5: 225 – 227. Sims, C.C. 1967.Graphs and Finite permutation group.Math.Zeitschrift95: 76 -86.
6. Sims, C. C. (1967). Graph and finite permutation groups. Math. Zeitschrift 95: 76-86.

Appendix I: Suborbits of the Alternating Group $G=A_5$ Acting on Ordered Triples

$Orb_{G_{[1,2,3]}}[1,2,3] = \{[1,2,3]\} = \Delta_0$, $Orb_{G_{[1,2,3]}}[1,3,2] = \{[1,3,2]\} = \Delta_1$, $Orb_{G_{[1,2,3]}}[1,2,4] = \{[1,2,4]\} = \Delta_2$, $Orb_{G_{[1,2,3]}}[1,4,2] = \{[1,4,2]\} = \Delta_3$, $Orb_{G_{[1,2,3]}}[1,2,5] = \{[1,2,5]\} = \Delta_4$, $Orb_{G_{[1,2,3]}}[1,5,2] = \{[1,5,2]\} = \Delta_5$, $Orb_{G_{[1,2,3]}}[1,3,4] = \{[1,3,4]\} = \Delta_6$, $Orb_{G_{[1,2,3]}}[1,4,3] = \{[1,4,3]\} = \Delta_7$, $Orb_{G_{[1,2,3]}}[1,3,5] = \{[1,3,5]\} = \Delta_8$, $Orb_{G_{[1,2,3]}}[1,5,3] = \{[1,5,3]\} = \Delta_9$, $Orb_{G_{[1,2,3]}}[1,4,5] = \{[1,4,5]\} = \Delta_{10}$, $Orb_{G_{[1,2,3]}}[1,5,4] = \{[1,5,4]\} = \Delta_{11}$, $Orb_{G_{[1,2,3]}}[2,1,3] = \{[2,1,3]\} = \Delta_{12}$, $Orb_{G_{[1,2,3]}}[2,3,1] = \{[2,3,1]\} = \Delta_{13}$, $Orb_{G_{[1,2,3]}}[2,1,4] = \{[2,1,4]\} = \Delta_{14}$, $Orb_{G_{[1,2,3]}}[2,4,1] = \{[2,4,1]\} = \Delta_{15}$, $Orb_{G_{[1,2,3]}}[2,1,5] = \{[2,1,5]\} = \Delta_{16}$, $Orb_{G_{[1,2,3]}}[2,5,1] = \{[2,5,1]\} = \Delta_{17}$,

$Orb_{G_{[1,2,3]}}[2,3,4] = \{[2,3,4]\} = \Delta_{18}$, $Orb_{G_{[1,2,3]}}[2,4,3] = \{[2,4,3]\} = \Delta_{19}$, $Orb_{G_{[1,2,3]}}[2,3,5] = \{[2,3,5]\} = \Delta_{20}$, $Orb_{G_{[1,2,3]}}[2,5,3] = \{[2,5,3]\} = \Delta_{21}$, $Orb_{G_{[1,2,3]}}[2,4,5] = \{[2,4,5]\} = \Delta_{22}$, $Orb_{G_{[1,2,3]}}[2,5,4] = \{[2,5,4]\} = \Delta_{23}$, $Orb_{G_{[1,2,3]}}[3,1,2] = \{[3,1,2]\} = \Delta_{24}$, $Orb_{G_{[1,2,3]}}[3,2,1] = \{[3,2,1]\} = \Delta_{25}$, $Orb_{G_{[1,2,3]}}[3,1,4] = \{[3,1,4]\} = \Delta_{26}$, $Orb_{G_{[1,2,3]}}[3,4,1] = \{[3,4,1]\} = \Delta_{27}$, $Orb_{G_{[1,2,3]}}[3,1,5] = \{[3,1,5]\} = \Delta_{28}$, $Orb_{G_{[1,2,3]}}[3,5,1] = \{[3,5,1]\} = \Delta_{29}$, $Orb_{G_{[1,2,3]}}[3,2,4] = \{[3,2,4]\} = \Delta_{30}$, $Orb_{G_{[1,2,3]}}[3,4,2] = \{[3,4,2]\} = \Delta_{31}$, $Orb_{G_{[1,2,3]}}[3,2,5] = \{[3,2,5]\} = \Delta_{32}$, $Orb_{G_{[1,2,3]}}[3,5,2] = \{[3,5,2]\} = \Delta_{33}$, $Orb_{G_{[1,2,3]}}[3,4,5] = \{[3,4,5]\} = \Delta_{34}$, $Orb_{G_{[1,2,3]}}[3,5,4] = \{[3,5,4]\} = \Delta_{35}$, $Orb_{G_{[1,2,3]}}[4,1,2] = \{[4,1,2]\} = \Delta_{36}$, $Orb_{G_{[1,2,3]}}[4,2,1] = \{[4,2,1]\} = \Delta_{37}$, $Orb_{G_{[1,2,3]}}[4,1,3] = \{[4,1,3]\} = \Delta_{38}$, $Orb_{G_{[1,2,3]}}[4,3,1] = \{[4,3,1]\} = \Delta_{39}$, $Orb_{G_{[1,2,3]}}[4,1,5] = \{[4,1,5]\} = \Delta_{40}$, $Orb_{G_{[1,2,3]}}[4,5,1] = \{[4,5,1]\} = \Delta_{41}$, $Orb_{G_{[1,2,3]}}[4,2,3] = \{[4,2,3]\} = \Delta_{42}$, $Orb_{G_{[1,2,3]}}[4,3,2] = \{[4,3,2]\} = \Delta_{43}$, $Orb_{G_{[1,2,3]}}[4,2,5] = \{[4,2,5]\} = \Delta_{44}$, $Orb_{G_{[1,2,3]}}[4,5,2] = \{[4,5,2]\} = \Delta_{45}$, $Orb_{G_{[1,2,3]}}[4,3,5] = \{[4,3,5]\} = \Delta_{46}$, $Orb_{G_{[1,2,3]}}[4,5,3] = \{[4,5,3]\} = \Delta_{47}$, $Orb_{G_{[1,2,3]}}[5,1,2] = \{[5,1,2]\} = \Delta_{48}$, $Orb_{G_{[1,2,3]}}[5,2,1] = \{[5,2,1]\} = \Delta_{49}$, $Orb_{G_{[1,2,3]}}[5,1,3] = \{[5,1,3]\} = \Delta_{50}$, $Orb_{G_{[1,2,3]}}[5,3,1] = \{[5,3,1]\} = \Delta_{51}$, $Orb_{G_{[1,2,3]}}[5,1,4] = \{[5,1,4]\} = \Delta_{52}$, $Orb_{G_{[1,2,3]}}[5,4,1] = \{[5,4,1]\} = \Delta_{53}$, $Orb_{G_{[1,2,3]}}[5,2,3] = \{[5,2,3]\} = \Delta_{54}$, $Orb_{G_{[1,2,3]}}[5,3,2] = \{[5,3,2]\} = \Delta_{55}$, $Orb_{G_{[1,2,3]}}[5,2,4] = \{[5,2,4]\} = \Delta_{56}$, $Orb_{G_{[1,2,3]}}[5,4,2] = \{[5,4,2]\} = \Delta_{57}$, $Orb_{G_{[1,2,3]}}[5,3,4] = \{[5,3,4]\} = \Delta_{58}$, $Orb_{G_{[1,2,3]}}[5,4,3] = \{[5,4,3]\} = \Delta_{59}$

,Appendix II Suborbits of the Alternating Group G=A_6 Acting on Ordered Triples

$Orb_{G_{[1,2,3]}}[1,2,3] = \{[1,2,3]\} = \Delta_0$, $Orb_{G_{[1,2,3]}}[1,3,2] = \{[1,3,2]\} = \Delta_1$, $Orb_{G_{[1,2,3]}}[2,1,3] = \{[2,1,3]\} = \Delta_2$, $Orb_{G_{[1,2,3]}}[2,3,1] = \{[2,3,1]\} = \Delta_3$, $Orb_{G_{[1,2,3]}}[3,1,2] = \{[3,1,2]\} = \Delta_4$, $Orb_{G_{[1,2,3]}}[3,2,1] = \{[3,2,1]\} = \Delta_5$, $Orb_{G_{[1,2,3]}}[1,4,5] = \{[1,4,5], [1,4,6]\} = \Delta_6$, $Orb_{G_{[1,2,3]}}[1,5,4] = \{[1,5,4], [1,6,4]\} = \Delta_7$, $Orb_{G_{[1,2,3]}}[4,5,1] = \{[4,5,1], [4,6,1]\} = \Delta_8$, $Orb_{G_{[1,2,3]}}[4,1,5] = \{[4,1,5], [4,1,6]\} = \Delta_9$, $Orb_{G_{[1,2,3]}}[5,4,1] = \{[5,4,1], [6,4,1]\} = \Delta_{10}$, $Orb_{G_{[1,2,3]}}[5,1,4] = \{[5,1,4], [6,1,4]\} = \Delta_{11}$, $Orb_{G_{[1,2,3]}}[2,4,5] = \{[2,4,5], [2,4,6]\} = \Delta_{12}$, $Orb_{G_{[1,2,3]}}[2,5,4] = \{[2,5,4], [2,6,4]\} = \Delta_{13}$, $Orb_{G_{[1,2,3]}}[4,2,5] = \{[4,2,5], [4,2,6]\} = \Delta_{14}$, $Orb_{G_{[1,2,3]}}[4,5,2] = \{[4,5,2], [4,6,2]\} = \Delta_1$ $Orb_{G_{[1,2,3]}}[5,2,4] = \{[5,2,4], [6,2,4]\} = \Delta_{16}$, $Orb_{G_{[1,2,3]}}[5,4,2] = \{[5,4,2], [6,4,2]\} = \Delta_{17}$, $Orb_{G_{[1,2,3]}}[3,4,5] = \{[3,4,5], [3,4,6]\} = \Delta_{18}$, $Orb_{G_{[1,2,3]}}[3,5,4] = \{[3,5,4], [3,6,4]\} = \Delta_{19}$, $Orb_{G_{[1,2,3]}}[4,3,5] = \{[4,3,5], [4,3,6]\} = \Delta_{20}$ $Orb_{G_{[1,2,3]}}[4,5,3] = \{[4,5,3], [4,6,3]\} = \Delta_{21}$, $Orb_{G_{[1,2,3]}}[5,4,3] = \{[5,4,3], [6,4,3]\} = \Delta_{22}$, $Orb_{G_{[1,2,3]}}[5,3,4] = \{[5,3,4], [6,3,4]\} = \Delta_{23}$, $Orb_{G_{[1,2,3]}}[1,2,4] = \{[1,2,4], [1,2,5], [1,2,6]\} = \Delta_{24}$, $Orb_{G_{[1,2,3]}}[2,1,4] = \{[2,1,4], [2,1,5], [2,1,6]\} = \Delta_{25}$, $Orb_{G_{[1,2,3]}}[1,4,2] = \{[1,4,2], [1,5,2], [1,6,2]\} = \Delta_{26}$, $Orb_{G_{[1,2,3]}}[2,4,1] = \{[2,4,1], [2,5,1], [2,6,1]\} = \Delta_{27}$, $Orb_{G_{[1,2,3]}}[4,2,1] = \{[4,2,1], [5,2,1], [6,2,1]\} = \Delta_{28}$, $Orb_{G_{[1,2,3]}}[4,1,2] = \{[4,1,2], [5,1,2], [6,1,2]\} = \Delta_{29}$, $Orb_{G_{[1,2,3]}}[1,3,4] = \{[1,3,4], [1,3,5], [1,3,6]\} = \Delta_{30}$, $Orb_{G_{[1,2,3]}}[3,1,4] = \{[3,1,4], [3,1,5], [3,1,6]\} = \Delta_{31}$, $Orb_{G_{[1,2,3]}}[1,4,3] = \{[1,4,3], [1,5,3], [1,6,3]\} = \Delta_{32}$, $Orb_{G_{[1,2,3]}}[3,4,1] = \{[3,4,1], [3,5,1], [3,6,1]\} = \Delta_{33}$, $Orb_{G_{[1,2,3]}}[4,1,3] = \{[4,1,3], [5,1,3], [6,1,3]\} = \Delta_{34}$, $Orb_{G_{[1,2,3]}}[4,3,1] = \{[4,3,1], [5,3,1], [6,3,1]\} = \Delta_{35}$, $Orb_{G_{[1,2,3]}}[2,3,4] = \{[2,3,4], [2,3,5], [2,3,6]\} = \Delta_{36}$, $Orb_{G_{[1,2,3]}}[3,2,4] = \{[3,2,4], [3,2,5], [3,2,6]\} = \Delta_{37}$, $Orb_{G_{[1,2,3]}}[2,4,3] = \{[2,4,3], [2,5,3], [2,6,3]\} = \Delta_{38}$, $Orb_{G_{[1,2,3]}}[3,4,2] = \{[3,4,2], [3,5,2], [3,6,2]\} = \Delta_{39}$, $Orb_{G_{[1,2,3]}}[4,2,3] = \{[4,2,3], [5,2,3], [6,2,3]\} = \Delta_{40}$, $Orb_{G_{[1,2,3]}}[4,3,2] = \{[4,3,2], [5,3,2], [6,3,2]\} = \Delta_{41}$, $Orb_{G_{[1,2,3]}}[4,5,6] = \{[4,5,6], [4,6,5], [5,4,6]\} = \Delta_{42}$,

$Orb_{G_{[1,2,3]}}$ [6,5,4] = {[6,5,4], [6,4,5], [5,6,4] } = Δ_{43}

Appendix III Suborbits of the Alternating Group G=A_7 Acting on Ordered Triples

$Orb_{G_{[1,2,3]}}$ [1,2,3] = {[1,2,3]} = Δ_0 , $Orb_{G_{[1,2,3]}}$ [1,3,2] = {[1,3,2]} = Δ_1 , $Orb_{G_{[1,2,3]}}$ [2,1,3] = {[2,1,3]} = Δ_2 , $Orb_{G_{[1,2,3]}}$ [2,3,1] = {[2,3,1]} = Δ_3 , $Orb_{G_{[1,2,3]}}$ [3,1,2] = {[3,1,2]} = Δ_4 , $Orb_{G_{[1,2,3]}}$ [3,2,1] = {[3,2,1]} = Δ_5 , $Orb_{G_{[1,2,3]}}$ [1,2,4] = {[1,2,4], [1,2,5], [1,2,6], [1,2,7] } = Δ_6 , $Orb_{G_{[1,2,3]}}$ [2,1,4] = {[2,1,4], [2,1,5], [2,1,6], [2,1,7] } = Δ_7 , $Orb_{G_{[1,2,3]}}$ [1,4,2] = {[1,4,2], [1,5,2], [1,6,2], [1,7,2] } = Δ_8 , $Orb_{G_{[1,2,3]}}$ [2,4,1] = {[2,4,1], [2,5,1], [2,6,1], [2,7,1] } = Δ_9 , $Orb_{G_{[1,2,3]}}$ [4,2,1] = {[4,2,1], [5,2,1], [6,2,1], [7,2,1] } = Δ_{10} , $Orb_{G_{[1,2,3]}}$ [4,1,2] = {[4,1,2], [5,1,2], [6,1,2], [7,1,2] } = Δ_{11} , $Orb_{G_{[1,2,3]}}$ [1,3,4] = {[1,3,4], [1,3,5], [1,3,6], [1,3,7] } = Δ_{12} , $Orb_{G_{[1,2,3]}}$ [3,1,4] = {[3,1,4], [3,1,5], [3,1,6], [3,1,7] } = Δ_{13} , $Orb_{G_{[1,2,3]}}$ [1,4,3] = {[1,4,3], [1,5,3], [1,6,3], [1,7,3] } = Δ_{14} , $Orb_{G_{[1,2,3]}}$ [3,4,1] = {[3,4,1], [3,5,1], [3,6,1], [3,7,1] } = Δ_{15} , $Orb_{G_{[1,2,3]}}$ [4,1,3] = {[4,1,3], [5,1,3], [6,1,3], [7,1,3] } = Δ_{16} , $Orb_{G_{[1,2,3]}}$ [4,3,1] = {[4,3,1], [5,3,1], [6,3,1], [7,3,1] } = Δ_{17} , $Orb_{G_{[1,2,3]}}$ [2,3,4] = {[2,3,4], [2,3,5], [2,3,6], [2,3,7] } = Δ_{18} , $Orb_{G_{[1,2,3]}}$ [3,2,4] = {[3,2,4], [3,2,5], [3,2,6], [3,2,7] } = Δ_{19} , $Orb_{G_{[1,2,3]}}$ [2,4,3] = {[2,4,3], [2,5,3], [2,6,3], [2,7,3] } = Δ_{20} , $Orb_{G_{[1,2,3]}}$ [3,4,2] = {[3,4,2], [3,5,2], [3,6,2], [3,7,2] } = Δ_{21} , $Orb_{G_{[1,2,3]}}$ [4,2,3] = {[4,2,3], [5,2,3], [6,2,3], [7,2,3] } = Δ_{22} , $Orb_{G_{[1,2,3]}}$ [4,3,2] = {[4,3,2], [5,3,2], [6,3,2], [7,3,2] } = Δ_{23} , $Orb_{G_{[1,2,3]}}$ [1,4,5] = {[1,4,5], [1,4,6], [1,4,7], [1,5,4], [1,5,6], [1,5,7], [1,6,4], [1,6,5], [1,6,7], [1,7,4], [1,7,5], [1,7,6] } = Δ_{24} , $Orb_{G_{[1,2,3]}}$ [4,1,5] = {[4,1,5], [4,1,6], [4,1,7], [5,1,4], [5,1,6], [5,1,7], [6,1,4], [6,1,5], [6,1,7], [7,1,4], [7,1,5], [7,1,6] } = Δ_{25} , $Orb_{G_{[1,2,3]}}$ [4,5,1] = {[4,5,1], [4,6,1], [4,7,1], [5,4,1], [5,6,1], [5,7,1], [6,4,1], [6,5,1], [6,7,1], [7,4,1], [7,5,1], [7,6,1] } = Δ_{26} , $Orb_{G_{[1,2,3]}}$ [2,4,5] = {[2,4,5], [2,4,6], [2,4,7], [2,5,4], [2,5,6], [2,5,7], [2,6,4], [2,6,5], [2,6,7], [2,7,4], [2,7,5], [2,7,6] } = Δ_{27} , $Orb_{G_{[1,2,3]}}$ [4,2,5] = {[4,2,5], [4,2,6], [4,2,7], [5,2,4], [5,2,6], [5,2,7], [6,2,4], [6,2,5], [6,2,7], [7,2,4], [7,2,5], [7,2,6] } = Δ_{28} , $Orb_{G_{[1,2,3]}}$ [4,5,2] = {[4,5,2], [4,6,2], [4,7,2], [5,4,2], [5,6,2], [5,7,2], [6,4,2], [6,5,2], [6,7,2], [7,4,2], [7,5,2], [7,6,2] } = Δ_{29} , $Orb_{G_{[1,2,3]}}$ [3,4,5] = {[3,4,5], [3,4,6], [3,4,7], [3,5,4], [3,5,6], [3,5,7], [3,6,4], [3,6,5], [3,6,7], [3,7,4], [3,7,5], [3,7,6] } = Δ_{30} , $Orb_{G_{[1,2,3]}}$ [4,3,5] = {[4,3,5], [4,3,6], [4,3,7], [5,3,4], [5,3,6], [5,3,7], [6,3,4], [6,3,5], [6,3,7], [7,3,4], [7,3,5], [7,3,6] } = Δ_{31} , $Orb_{G_{[1,2,3]}}$ [4,5,3] = {[4,5,3], [4,6,3], [4,7,3], [5,4,3], [5,6,3], [5,7,3], [6,4,3], [6,5,3], [6,7,3], [7,4,3], [7,5,3], [7,6,3] } = Δ_{32} , $Orb_{G_{[1,2,3]}}$ [4,5,6] = {[4,5,6], [4,5,7], [4,6,5], [4,6,7], [4,7,5], [4,7,6], [5,4,6], [5,4,7], [5,6,4], [5,6,7], [5,7,4], [5,7,6] } = Δ_{33} , $Orb_{G_{[1,2,3]}}$ [6,4,5] = {[6,4,5], [6,4,7], [6,5,4], [6,5,7], [6,7,4], [6,7,5], [7,4,5], [7,4,6], [7,5,4], [7,5,6], [7,6,4], [7,6,5] } = Δ_{34}