(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: **MAA 326**

COURSE TITLE: **ODE II**

DATE: 01/09/2022 **TIME:** 9 AM -11 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

(a) State the existence of uniqueness theorem of a system of differential equation. (2 mks)

(b) Replace the equation by a system of first order (4 mks)

$y^{III} - 3y^{II} + y^{I} - 6y = 2x$

(c) Use elimination method to solve the system (8 mks)

$$\frac{dx}{dt} + y = cost$$

$$\frac{dy}{dt} + 4x = sint$$

(d) Find the general solution of a system of differential equation. (8 mks)

$$X^{I} = \begin{pmatrix} 12 & -9 \\ 4 & 0 \end{pmatrix} X$$

(e) Compute e^{At} given that $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ (8 mks)

QUESTION TWO (20 MARKS)

(a) (i) Define the Gamma function (2 mks)

(ii) Show that $\Gamma(x+1) = x\Gamma(x)$ (4 mks)

(b) Consider the system of differential equations

$$\frac{dx}{dt} = 2x - y - z$$

$$\frac{dy}{dt} = -x + 2y - z$$

$$\frac{dz}{dt} = -x - y + 2z$$

(i) Find the general solution to the system (10 mks)

(ii) Find the particular solution given the initial value $x(0) = 0$, $y(0) = 0$, $z(0) = 0$ (4 mks)

QUESTION THREE (20 MARKS)

(a) Solve $X^{I} = AX + B(t)$ for

$A = \begin{bmatrix} 2 & 1 \\ -4 & 2 \end{bmatrix}$; $B(t) = \begin{bmatrix} 3e^{2t} \\ te^{2t} \end{bmatrix}$ (10 mks)

(b) Apply Picard's method to solve the following initial value problem up to 3rd approximation

$\frac{dy}{dx} = 2x + y, y(0) = 3$ (10 mks)

QUESTION FOUR (20 MARKS)

(a) Define the term Bifurcation (2 mks)

(b) State the condition for the following critical points to occur and in each case draw the phase portrait

i) A node (4 mks)
ii) Centre (4 mks)
iii) Saddle point (2 mks)

(c) Consider two competing species living in an ocean. Let $x(t)$and $y(t)$ denote respective population of the species at a time(t). Suppose the initial populations are $x(0) = 300, y(0) = 100$, if the growth rate of the species are given by;

$\frac{dx}{dt} = -3x + 6y$ and $\frac{dy}{dt} = x + 2y$ find the population of each species at time t.

(8 Marks)

QUESTION FIVE (20 MARKS)

(a) Verify that on the interval $(-\infty, \infty)$ $X_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-2t}$ and $X_2 = \begin{pmatrix} 3 \\ 5 \end{pmatrix} e^{6t}$ are fundamental solutions of $X^{I} = \begin{pmatrix} 1 & 3 \\ 5 & 3 \end{pmatrix} X$ (9 mks)

(b) Consider the system

$$\frac{dx_1}{dt} = x_1 + 4x_2 + e^{x_1} - 3$$

$$\frac{dx_2}{dt} = -x_2 - x_2e^{x_1} - 4$$

(i) Linearize the system at the (0,0) (5 mks)

(ii) Solve the linearized system (6 mks)