(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **MAP 421**

COURSE TITLE: **TOPOLOGY II**

DATE: 01/09/2022 **TIME:** 9:00 AM - 11:00 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

a) Define the following terms
   (i). A compact space X. (2 mks)
   (ii). A unit sphere in $\mathbb{R}^n$ (2 mks)
   (iii). A second countable space (2 mks)
   (iv). A normal space (2 mks)
   (v). A T_1 space (2 mks)

b) Let $f: X \rightarrow Y$ be a continuous function from a topological space, X, to another, Y. Show that its image is compact if X is compact. (6 mks)

c) (i).Show that the subspace $Y = [0,1]$ of a real line is connected. (4 mks)
   (ii). Give an example of a subspace of Y that is not connected (1 mk)

d) What do you understand by the term a complete regular space? Give an example.(4 mks)

e) Prove that every subspace of a second countable space is a second countable (5 mks)

QUESTION TWO (20 MARKS)

a) Show that the interval $B = (0,1)$ of the real line with the usual topology is not sequentially compact. (5 mks)

b) Let $\{A_i\}i \in I$ be a collection of connected subspaces with a common point. Show that $\bigcup_{i\in I} A_i$is connected. (5 mks)

c) Prove that every open covering of a space X with a countable basis contains a countable sub collection covering X. (6 mks)

d) Find the smallest compact set containing (p, q) given that $p, q \in \mathbb{R}$. Can there be a separation of the new set determined? Explain? (4 mks)

QUESTION THREE (20 MARKS)

a) What is a linear continuum? (2 mks)

b) Let $I \times I$ be a product topological space and π_1, π_2 be projections on I respectively be defined as $\pi_1(x, y) = x$ and $\pi_2(x, y) = y$ for $x, y \in I$. Let $A \subset I \times I$ be square $A = \{x, y: a \leq x \leq b, c \leq y \leq d, a, b, c, d \in \mathbb{R}\}$. Show that A is a linear continuum.(8 mks)

c) State and prove the generalization of the intermediate value theorem. (10 mks)

QUESTION FOUR (20 MARKS)

a) Show that the space $\mathbb{R}_l$ is normal. (5 mks)

b) Let X be a topological space. Define a relation $x \sim y$ on X if there is a connected subspace of X containing both x and y. Show that $\sim$ is an equivalence relation. (6 mks)

c) When is a collection of subsets of a space X said to have a finite intersection property?

d) Define a set X as $X = \{a, b\}$. For a T_1 topological space from the set. Show that the space formed is a topological space but not T_1. (7 mks)

QUESTION FIVE (20 MARKS)

a) Define a T_2 space given an example. (3 mks)

b) What is path-connected space? Give an example (3 mks)

c) Define the term a compact space, hence show that the space of real numbers, $\mathbb{R}$, is not compact. (5 mks)

d) Prove that every metrizable space is normal. (9 mks)