Spaces of compact operators and their dual spaces

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Abstract

The ω'-topology on the space $L(X, Y)$ of bounded linear operators from the Banach space X into the Banach space Y is discussed in [10]. Let $\mathcal{L}^{w'}(X, Y)$ denote the space of all $T \in L(X, Y)$ for which there exists a sequence of compact linear operators $(T_n) \subset K(X, Y)$ such that $T = \omega' - \lim_n T_n$ and let $|||T||| := \{\sup_n \|T_n\| : T_n \in K(X,Y), T_n \to w' T\}$. We show that $(L^{w'}, |||\cdot|||)$ is a Banach ideal of operators and that the continuous dual space $K(X, Y)^*$ is complemented in $(L^{w'}(X,Y), |||\cdot|||)^*$. This results in necessary and sufficient conditions for $K(X, Y)$ to be reflexive, whereby the spaces X and Y need not satisfy the approximation property. Similar results follow when X and Y are locally convex spaces.