(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **MAA 423/MAT 426**

COURSE TITLE: **FOURIER SERIES**

DATE: 05/09/2022 **TIME:** 9:00 AM - 11:00 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION 1 (30 MARKS)

a) Find the limit $\lim_{x\to 0}\left\{\dfrac{\cos(x^6)-1+\frac{1}{12}x^{10}}{x^8}\right\}$ [5 Marks]

b) Determine whether the given functions are even, odd or neither

i) $f(x)=\sin\left(\dfrac{n\pi x}{L}\right)$ on $-L\le x\le L$ [2 Marks]

ii) $f(x)=\cos\left(\dfrac{n\pi x}{L}\right)$ on $-L\le x\le L$ [2 Marks]

iii) $f(x)=x^2+e^{6-x}$ on $-2\le x\le 2$ [3 Marks]

c) Compute the Maclaurin series as far as x^6 term for the following functions

e^{-x} , $x\sin(x)$, , $\dfrac{\cos(x^2)}{x^2}$ [2 Marks]

d) The Fourier series for the function f defined by $f(x)=x^2$ on the interval $[-\pi,\pi]$ is known to be convergent. What do you understand by

i) period of f [3 Marks]

ii) f is periodic

iii) periodic extension of f [2 Marks]

iv) Fourier coefficients of expansion of f [3Marks]

e) Give a sketch graph of three periodic extensions of f defined by $f(x)=x^2$ on the interval $[-10,10]$ [8 Marks]

QUESTION 2 (20MARKS)

One cycle of a periodic waveform $y=f(x)$ of period 2π is defined by the below data.

Assume that $f(x)$ has a uniformly convergent Fourier series with

$$f(x)\sim\frac{a_0}{2}+\sum_{n=1}^{\infty}\{a_n\cos(nx)+b_n\sin(nx)\};\quad 0\le x\le 2\pi .$$

Applying harmonic analysis

a) Determine the approximate function $Y=\hat{f}$ Fourier series for $y=f(x)$ up to and including the 3rd harmonic. [12 Marks]

b) Predict $Y=\hat{f}(20^0)$, $Y=\hat{f}(200^0)$ [8 Marks]

x^0	0	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240
$y(x)$	15	17	20	22	23	23.4	24	21	20	15	8	5	3	3.2	4	6	9

x^0	255	270	285	300	315	330	345
$y(x)$	10.3	11	11.6	10	10.8	11	11.5

QUESTION 3 (20MARKS)

a) Solve the heat equation $u_t = \alpha^2 u_{xx}$, $0 < x < 1$, $t > 0$ with the Dirichlet boundary conditions $u(t,0) = u(t,1) = 0$, $t > 0$ and initial conditions $u(0,x) = g(x) = x^2$, $0 \le x \le 1$ [8 Marks]

b) Solve the differential equation $y'' - 144y = \begin{cases} x & ;0<x<5 \\ -x & ;5\le x<10 \end{cases}$, [12 Marks]

QUESTION 4 (20MARKS)

Given real valued function $y = f(x)$ for which

$$f(x) = \begin{cases} 0 & -\pi < x < \pi \\ -x & 0 < x < \pi \end{cases}$$

$$f(x) = f(x+2\pi)$$

(a)Sketch the graph of $f(x)$ over the interval $-4\pi < x < 4\pi$ [8 marks]

(b)State period of $f(x)$ [3 marks]

(c)Deduce that

$$f(x) = \frac{\pi}{4} + \frac{2}{\pi}\left\{\cos x + \frac{1}{9}\cos 3x + \frac{1}{25}\cos 5x + \frac{1}{49}\cos 7x...\right\} + \sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x - \frac{1}{4}\sin 4x...$$

[9 marks]

QUESTION 5 (20MARKS)

(a) Consider the function defined on $[0, \pi)$ by

$$f(x) = \begin{cases} x & 0 \le x < \frac{\pi}{2} \\ \pi - x & \frac{\pi}{2} \le x < \pi \end{cases}$$

Obtain the Fourier **half-range** sine for this function $f(x)$. [14 marks]

Sketch the anti-symmetric odd periodic extension of f(x) on (- π , π)

(b) Let $f(x)$ be periodic function with period 2π with

$$f(x) = \sin^2 x, \; -\pi \le x \le \pi$$

(i) State giving reasons whether $f(x)$ is odd or even.

(ii) Sketch $f(x)$ over the interval $[-16\pi, \; 16\pi]$

(iii)Obtain the Fourier series expansion for $f(x)$ [6 marks]