(Knowledge for Development)

# KIBABII UNIVERSITY

## UNIVERSITY EXAMINATIONS

## 2021/2022 ACADEMIC YEAR

## FOURTH YEAR SECOND SEMESTER

## MAIN EXAMINATION

## FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

**COURSE CODE:** MAT 422/MAA 421

**COURSE TITLE:** PDE II

**DATE:** 01/09/2022 **TIME:** 2:00 PM - 4:00 PM

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## INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

## QUESTION ONE COMPULSORY (30 MARKS)

(a) Define the following terms giving an example in each;

i. Reducible linear differential operator.

ii. Linear non-homogeneous partial differential equation. (4 marks)

(b) Using method of separation of variables, solve

$\frac{\partial u}{\partial x} = 3\frac{\partial u}{\partial t} + u$ where $u(x, 0) = 6e^{-5x}$ (6 marks)

(c) Using method of characteristics solve the semi-linear partial differential equation.

$-x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = u$ given $u = x^3$ on $y = x, 1 \leq y \leq 2$ (8 marks)

(d) Determine the complete solution of

i. $\frac{\partial^3 z}{\partial x^3} - 7\frac{\partial^3 z}{\partial x \partial y^2} - 6\frac{\partial^3 z}{\partial y^3} = \sin(x + 2y)$ (6 marks)

ii. $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} - 3\frac{\partial z}{\partial x} + 3\frac{\partial z}{\partial y} = e^{2x+3y}$ (6 marks)

## QUESTION TWO (20 MARKS)

(a) A flexible string of length 2 units is fixed on the $x - axis$ at $x = 0$ and $x = 2$. The tension per mass per unit length of the string is given by 36. The string is given by an initial displacement of $y(x, 0) = 6\sin(\pi x) - 3\sin\left(\frac{7\pi x}{2}\right)$. The string is gently released from rest to oscillate. Write down the mathematical model of this motion to include the PDE, boundary and initial conditions, then find the displacement wave which depends on $x$ and $t$ at any time $t$. (15 marks)

(b) Using method of characteristics solve the semi-linear partial differential equation.

$4\frac{\partial u}{\partial y} - 2\frac{\partial u}{\partial x} + 5u = 0$ (5 marks)

## QUESTION THREE (20 MARKS)

(a) Classify the given PDE below and determine its characteristic curves

$$y^2\frac{\partial^2 u}{\partial x^2} - 2xy\,\frac{\partial^2 u}{\partial x\partial y} + x^2\frac{\partial^2 u}{\partial y^2} = y^3\frac{\partial u}{\partial x} + x^4\frac{\partial u}{\partial y}$$ (3 marks)

(b) Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial x} + 3\frac{\partial z}{\partial y} - 2z = x^2y^2$ (11 marks)

(c) Solve the wave equation by D'Alembert's method

$\frac{\partial^2 y}{\partial t^2} = C^2\frac{\partial^2 y}{\partial x^2}$ where C is a constant (6 marks)

## QUESTION FOUR (20 MARKS)

Given the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} - 4\frac{\partial^2 u}{\partial x\partial y} + 4\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial x} - 2\frac{\partial u}{\partial y} = 0$$

i. Classify the PDE (2 marks)

ii. Reduce it to canonical form and solve it (18 marks)

## QUESTION FIVE (20 MARKS)

(a) Differentiate between Laplace's equation and Poisson's equation. (2 marks)

(b) Find the temperature function $u(x, t)$ on an insulated metallic rod of length $L$ which is governed by $\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$ under the conditions $u(0,t) = 0, u(L,t) = 0$and $u(L,t) = \frac{200\,x}{L}$ (13 marks)

(c) Solve $D(D^2 + D')(D + 4D')z = 0$ (5 marks)