(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREES OF BACHELOR OF SCIENCE

COURSE CODE: STA 422

COURSE TITLE: SEQUENTIAL ANALYSIS

DATE: 01/09/2022 **TIME: 9:00 AM - 11:00 AM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) Define the term *Sequential Analysis.* (2 Marks)

b) Briefly elaborate 2 applications of sequential analysis (4 Marks)

c) List the 3 key decision rules that are in sequential testing (3 Marks)

d) Let B_n denote the subset of n-dimensional space in which $A < \ell_k(\mathcal{E}_1, \ldots, \mathcal{E}_k) < B$ for $k = 1,2,\ldots, n-1$ and $\ell_k(\mathcal{E}_1, \ldots, \mathcal{E}_n) \geq B$ so that $\{N = n, \ell_n \geq B\} = \{(x_1, \ldots x_n) \in B_n\}$. Show that $\alpha \approx \frac{1-A}{B-A}$ and $\beta \approx \frac{A(B-1)}{B-A}$ (6 Marks)

e) Let $x_1, \ldots x_n$ be independent and identically distributed random variables with finite mean μ. Let M be any integer-valued random variable such that $\{M = n\}$ is an event determined only by $x_1, \ldots x_n$ for all $n=1,2,\ldots,$ and assume that $E(M) < \infty$ through monotone convergence theorem prove the Wald's equation (6 Marks)

f) Let θ be the probability of an item being defective. At the n^{th} stage, take one more observation if $B < \frac{\theta_1^r(1-\theta_1)^{n-r}}{\theta_0^r(1-\theta_0)^{n-r}} < A$. If $\theta_0 = 0.5$ and $\theta_1 = 0.8$, solve for A and B and hence determine the continue-sampling region. (5 Marks)

g) If the probability that an individual will suffer a bad reaction from injection of a given serum is 0.001, determine the probability that out of 2000 individuals,

(i) exactly 3, (2 Marks)

(ii) More than 2, individuals will suffer a bad reaction. Assume *X* is Poisson distributed (2 Marks)

QUESTION TWO (20 MARKS)

a) Consider the Problem of testing $\theta = \theta_0$ versus $\theta = \theta_1 > \theta_0$ in a Bernoulli population.

i. Derive the equation for θ (5 Marks)

ii. If $\theta_1 = 0.8,\ \theta_0 = 0.5\ and\ \alpha = \beta = 0.01$ compute the values of θ and Operating Characteristic function in the table below. (5 Marks)

h	-∞	-1	0	1	∞
θ					
OC					

b) By Wald's likelihood ratio theorem derive the stopping time inequality of any sequential hypothesis. (10 Marks)

QUESTION THREE (20 MARKS)

a) The sample size needed to reach a decision in a sequential or a multiple sampling plan is a random variable N. Assuming $P((Z = 0) < 1)$ show that the moment-generating function of N is finite and hence derive the expectation equation of this distribution.(10 Marks)

b) Using Wolfowitz method show that $E(1n(\Lambda_N) = E(N)E(Z)$ (10 Marks)

QUESTION FOUR (20 MARKS)

a) The number of miles an automobile tire lasts before it reaches a critical point in tread wear can be represented by a pdf

$$f(x) = \begin{cases} \frac{1}{30}e^{-\frac{x}{30}}, & for\ x > 0 \\ 0, & otherwise \end{cases}$$

Find the expected number of miles (in thousands) a tyre would last until it reaches the critical tread wear point. (10 Marks)

b) Prove the (weak) law of large numbers for Bernoulli trials by Chebyshev's inequality (10 Marks)

QUESTION FIVE (20 MARKS)

a) A function h(q) is estimable unbiasedly if and only if it can be expanded in Taylor's series in the interval $|q| < 1$. Prove that if h(q) is estimable, then its unique unbiased estimator is given by

$$g(\gamma_k) = \frac{(c-1)!}{(k+c-1)!}\frac{d^k}{dq^k}\left[\frac{h(q)}{(1-q)^c}\right]_{q=0}, k = 0,1,2, \ldots$$

(10 Marks)

b) Let $\theta = (\sigma^*/\sigma)^2$. Then as n gets large, in probability

$$\frac{M\theta}{n_1} \to \begin{cases} 1 & when\ H_0\ is\ true \\ 1 + \frac{\delta^{*2}}{4\sigma^2} & when\ \mu_2 - \mu_1 = \delta^* \end{cases}$$

Show that $\sigma^* = T_1 + T_2 = \alpha$ for all values of θ (10 Marks)