Question 1(30 marks)

a. Compute the eigenvalues and eigenvectors of $A = \begin{bmatrix} 3 & 2 \\ 0 & -1 \end{bmatrix}$ (7 markS)

b. Solve the following system of linear equations (10 marks)

$$2x_1 + x_2 + 4x_3 = 12$$
$$8x_1 - 3x_2 + 2x_3 = 20$$
$$4x_1 + 4x_2 - 3x_3 = 33$$

Using the following LU decomposition methods

i. Doolittle
ii. Cholesky

c. Solve the following system of equations using Jacobi iterative method given $x_0 = y_0 = z_0 = 0$ (iterations) (8 marks)

$$4x + y + 3z = 17$$
$$x + 5y + z = 14$$
$$2x - y + 8z = 12$$

d. Proof that $T_0(x) = 1$ and $T_1(x) = x$ in Chebyshev polynomials (5 marks)

Question 2(20 marks)

a. Find the first 3 non zero terms of Taylor series of the following equation $\frac{dy}{dx} = -xy$ given $y(0) = 1$ hence find $y(0.1)$ (12 marks)

b. Derive the recurrence relation of Chebyshev polynomial of first order and find the first five polynomial equations (8 marks)

Question 3 (20 marks)

a. Obtain the cubic spline approximation for the following set of data

x	1	2	3	4
F(x)	1	5	11	8

Hence interpolate at $Y(1.5)$ and $Y'(2)$ (15 marks)

b. Solve the following linear systems of equation using Successive over relaxation method (SOR) with $w = 1.25$ and $x^0 = (1,1,1)$ (3 iterations) (5 marks)

$$4x_1 + 3x_2 = 24$$
$$3x_1 + 4x_2 - x_3 = 30$$
$$-x_2 + 4x_3 = -24$$

Question 4 (20 marks)

a. If $A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$ $P = \begin{bmatrix} 1 & 1 & 3 \\ 0 & 3 & 2 \\ 1 & 1 & 1 \end{bmatrix}$ show that $P^{-1}AP = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (10 marks)

b. Find the eigenvalues and eigenvectors of the following matrix (10 marks)

$$A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

Question 5 (20 marks)

a. Evaluate $\int_0^1 \frac{1}{1+x} dx$ using Gauss Legendre (i) 2 point (ii) 3 point and (iii) 4 point formula (12 marks)

b. Evaluate the integral $\int_{-1}^{1} (1-x^2)^{\frac{3}{2}} cosx \, dx$ using Gauss Chebyshev intergral method point 1 and 2 (8 marks)