(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: STA 423

COURSE TITLE: BAYESIAN STATISTICS

DATE: 30/08/2022 **TIME:** 9:00 AM - 11:00 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)(COMPULSORY)

1. (a) State and explain any two differences between frequentist and Bayesian statistics (4 mks)

   (b) Give an expression for the Bayes rule for the conditional $P(y|x)$ and state the importance of the denominator (Assume X and Y are continuous random variables) (3 mks)

   (c) In order to determine how effective a magazine is reaching its target audience, a market research company selects a random sample of people from the target audience and interviews them. Out of 150 people in the sample, 29 had seen the latest issue.

      i. What is the distribution of y, the number who have seen the latest issue? (2 mks)

      ii. Use the uniform prior for π, the proportion of the target audience that has seen the latest issue. What is the posterior distribution of π? (3 mks)

   (d) Let $X \sim B(n, p)$. Assume the prior distribution of p is uniform on [0,1]. Show that the posterior is essentially the likelihood function. (5 mks)

   (e) An urn with a total of 8 balls, some of which are red and the rest of which are green. Let the random variable X be the number of red balls in the urn. Find the Bayesian estimate of X (5 mks)

   (f) A Bernoulli random variable X with a probability θ for success is known to be either 0.3 or 0.6. It is desired to test the null hypothesis $H_0 : \theta = 0.3$ against the alternative $H_1 : \theta = 0.6$ using the Bayes 0.05 test assuming the vague prior probability distribution for θ: $P(\theta = 0.3) = P(\theta = 0.6) = 0.5$. A sample of 30 trials on X yields 16 successes. Find the posterior probability of the null hypothesis (5 mks)

   (g) A bag contains two unbiased coins and a biased coin with probability for heads $P(H) = 0.3$. A coin is chosen at random from the bag and tossed three times. If two heads and a tail are obtained, what is the probability of the event that the coin is biased. (3 mks)

QUESTION TWO (20 MARKS)

2. (a) The cross-fertilized plant produce taller offspring than the self-fertilized plants. In order to obtain an estimate on the proportion θ of cross-fertilized that are taller, a statistical researcher observes a random sample of 15 pairs of plants that are exactly the same age. Each pair is grown in the same conditions with some cross-fertilized and others are self-fertilized. Based on previous experience, the researcher believes that the following are possible values of θ and that the prior probability for each value of θ is $p(\theta)$

θ	0.70	0.72	0.74	0.76	0.78	0.80
$p(\theta)$	0.12	0.16	0.23	0.24	0.15	0.10

   i. Find the $E(\theta)$ based on the posterior probabilities (7 mks)
   ii. Find the Bayesian estimate for the informative prior (4 mks)
   iii. Comment on the Bayesian estimate in (i) and (ii) above (3 mks)

   (b) A box contains two fair coins and a biased coin with probability for heads $P(H) = 0.2$. A coin is chosen at random from the box and tossed three times. If two heads and a tail are obtained, what is the probability of the event F, that the chosen coin is fair, and what is the probability of the event B, that the coin is biased? (6 mks)

QUESTION THREE (20 MARKS)

3. (a) The random variable X has a Poisson distribution with unknown parameter λ. It has been determined thay λ has the subjective prior probability function given in the table below. A random sample of size 3 yields the X-values 2,0, and 3.

λ	1.0	1.5	2.0
$\pi(\lambda)$	1/3	1/2	1/6

   i. Identify the prior used in this problem (2 mks)

ii. Find the likelihood of the data in $f(x|\lambda)$ (2 mks)

iii. Find the posterior distribution of λ (10 mks)

(b) Suppose $X_1, ..., X_n$ is a sample from geometric distribution with parameter p, $0 \leq p \leq 1$. Assume that the prior distribution of p is beta with $a = 4$ and $b = 4$. Find

i. the posterior distribution of p (3 mks)

ii. the Bayes estimate under quadratic loss function (3 mks)

QUESTION FOUR (20 MARKS)

4. (a) Suppose that X is a geometric random variable, where $P_X(k|\theta) = (1-\theta)^{k-1}\theta, k = 1, 2, \ldots$. Assume that the prior distribution for θ is the beta p.d.f. with parameters α and β. Find the posterior distribution for θ. (6 mks)

   (b) Suppose the binomial pdf describes the number of votes a candidate might receive in a poll conducted before the general election. Moreover, suppose a beta prior distribution has been assigned to θ, and every indicator suggests the election will be close. The pollster, then, has good reason for concentrating the bulk of the prior distribution around the value $\theta = \frac{1}{2}$. Setting the two beta parameters r and s both equal to 135 will accomplish that objective (in the event $r = s = 135$, the probability of θ being between 0.45 and 0.55 is approximately 0.90).

      i. Find the corresponding posterior distribution. (3 mks)

      ii. Find the squared-error loss Bayes estimate for θ and express it as a weighted average of the maximum likelihood estimate for θ and the mean of the prior pdf. (3 mks)

   (c) X is a Bernoulli random variable with success probability θ, which is known to be either 0.3 or 0.6. It is desired to test the null hypothesis $H_0 : \theta = 0.3$ against the alternative $H_1 : \theta = 0.6$ using a Bayes 0.05 test assuming the vague prior probability distribution for θ: $P(\theta = 0.3) = P(\theta = 0.6) = 0.5$. A sample of 30 trials on X yields 16 successes.

      i. Find the posterior probability of the null hypothesis (5 mks)

      ii. Check the rejection criterion of the Bayes 0.05 test (3 mks)

QUESTION FIVE (20 MARKS)

5. (a) A sample of 100 measurements of the diameter of a sphere gave a mean inch $\bar{x} = 4.38$. Based on prior experience, we know that the diameter is normally distributed with unknown mean and variance 0.36. Determine the posterior density of assuming a normal prior density with mean 4.5 inch and variance 0.4. (10 mks)

   (b) A student taking a standardized test is classified as gifted if he or she scores at least 100 out of a possible score of 150. Otherwise the student is classified as not gifted. Suppose the prior distribution of the scores of all students is a normal with mean 100 and standard deviation 15. It is believed that scores will vary each time the student takes the test and that these scores can be modeled as a normal distribution with mean μ and variance 100. Suppose the student takes the test and scores 115. Test the hypothesis that the student can be classified as a gifted student. (10 mks)