# KIBABII UNIVERSITY

## MAIN UNIVERSITY EXAMINATIONS

## ACADEMIC YEAR 2021/2022

## THIRD YEAR SECOND SEMESTER EXAMINATIONS

## BACHELOR OF SCIENCE

**COURSE CODE: SPC 323**

**COURSE TITLE: MATHEMATICAL PHYSICS II**

**DATE:** **05/09/2022** **TIME: 2:00PM-4:00PM**

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**INSTRUCTIONS TO CANDIDATES**

**Answer question ONE and any TWO of the remaining.**

**Time: 2 hours**

KIBU observes ZERO tolerance to examination cheating

## QUESTION ONE (30 MARKS)

a) Define the following terms:

   (i) A singular point of the function f(z) (2 marks)

   (ii) *A* Pole (2 marks)

   (iii) Residue (2 marks)

b) Using Cauchy's integral formula, evaluate $I=\oint_C \frac{dz}{z(z+2))}$ with C on a unit circle (3 marks)

c) Compute Laurent expansion about $Z_o=0$ at $f(z)=\frac{1}{z(z-1)}$ (3 marks)

d) Use Laplace transforms to evaluate, $f(t)=\cosh(kt)=\frac{1}{2}\left(e^{kt}+e^{-kt}\right)$ (3 marks)

e) Find the Fourier transform of $f(t)=e^{-\alpha|1|}$ where $\alpha>0$, (3 marks)

f) Use the Fourier integral to prove that (4 marks)

$$\int_0^\infty \frac{\cos axdx}{1+a^2}=\frac{\pi}{2}e^{-x}$$

g) Determine the residues of the following functions at the poles z=1 and z=-2 (4 marks)

$$\frac{1}{(z-1)(z+2)^2}$$

h) Find the Laurent series about the singularity for the function: (4 marks)

$$\frac{e^Z}{(z-2)^2}$$

## QUESTION TWO (20 MARKS)

a) Consider the chain decay in radioactivity $A\xrightarrow{\lambda_A}B\xrightarrow{\lambda_B}C$ where $\lambda_A$ and $\lambda_B$ are the disintegration constants. The equations of for the radioactive decays are:

$$\frac{dN_A(t)}{dt}=-\lambda_A N_A(t), \text{ and } \frac{dN_B(t)}{dt}=-\lambda_2 N_B(t)+\lambda_A N_A(t)$$

Where $N_A(t)$ and $N_B(t)$ are the number of atoms of A and B at time t, with initial conditions $N_A(A)=N_A^0;\ N_B(0)=0$. Apply Laplace transform to obtain $N_A(t)$ and $N_B(t)$, the number of atoms of A and B as a function of time t, in terms of $N_A^0$, $\lambda_A$ and $\lambda_B$ (10 marks)

b) Consider the radioactive decay:

$A \xrightarrow{\lambda_A} B \xrightarrow{\lambda_B} C$ (stable)

The equations for the chains are:

$$\frac{dN_A}{dt} = -\lambda_A N_A \quad (1)$$

$$\frac{dN_B}{dt} = -\lambda_B N_B + \lambda_A N_A \quad (2)$$

$$\frac{dN_C}{dt} = +\lambda_B N_B \quad (3)$$

with initial conditions $N_A(0) = N_A^0$; $N_B(0) = 0$ ; $N_C(0) = 0$ , where various symbols have usually meaning. Apply Laplace transforms to find the growth of C. (10 marks)

## QUESTION THREE (20 MARKS)

a) The Bessel function $J_n(x)$ is given by the series expansion (7 marks)

$$J_n(x) = \frac{\sum(-1)^k (x/2)^{n+2k}}{k!\Gamma(n+k+1)}$$

Show that $\frac{d}{dx}\left[x^n J_n(x)\right] = x^n J_{n-1}(x)$

b) Given that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$, obtain the formulae:

(i) $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x$ (7 marks)

(ii) $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cos x$ (6 marks)

## QUESTION FOUR (20 MARKS)

a) Show that the Legendre polynomials have the property (10 marks)

$$\int_{-l}^{l} P_n(x)P_m(x)\,\mathrm{d}x = \frac{2}{2n+1}, \text{ if } m = n$$
$$= 0, \text{ if } m \neq n$$

b) Show that for large n and small θ, $p_n(\cos\theta) \approx J_0(n\theta)$ (10 marks)

## QUESTION FIVE (20 MARKS)

a) Develop the Fourier series expansion for the saw-tooth (Ramp) wave f(x)=x/L, -L <x< L as in Figure 1 below (10 marks)

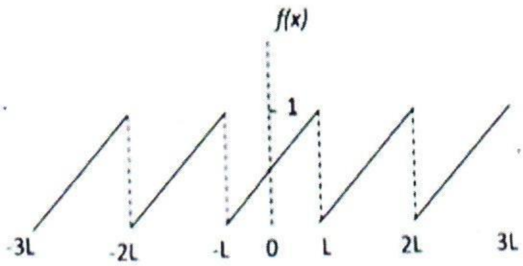


Figure 1

b) Find the Fourier transform of : (10 marks)

$$f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$$