23

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS
2021/2022 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER
MAIN EXAMINATIONS

FOR THE DEGREE OF BACHELOR OF SCIENCE (PHYSICS)

COURSE CODE: SPC 321

COURSE TITLE: QUANTUM MECHANICS I

DATE: 21 01/09/2022 **TIME:** ~~2:00PM~~-4:00PM 9.00

INSTRUCTIONS TO CANDIDATES

TIME: 2 HOURS

Answer question ONE and any TWO of the remaining

KIBU observes ZERO tolerance to examination cheating

QUESTION ONE [30 MARKS]

a) Prove that if the commutator $[\hat{A}, \hat{B}] = 1$ then:- $[\hat{A}, \hat{B}^2] = 2\hat{B}$. [3 marks]

b) If the operator $\frac{d^2}{dx^2}$ operates on the wave function $\psi = A\sin mx$ find the corresponding eigen value. [3 marks]

c) The state of a free particle is described by the following wave function:- [4 marks]

$$\psi(x) = \begin{cases} 0 \; for \; x < -3a \\ C \; for -3a < x < a \\ 0 \; for \; x > a \end{cases}$$

Find C and the probability of finding the particle in interval $[0, a]$.

d) A particle moving in x-axis has a wave function given by:- [3 marks]

$\psi(x) = \begin{cases} ax \; 0 \le x \le 1 \\ 0 \; elsewhere \end{cases}$ find $\langle x \rangle$

e) The one- dimensional time-independent Schrödinger equation is given by $\left(-\frac{\hbar^2}{2m}\right)\left(\frac{d^2\psi(x)}{dx^2}\right) + V(x)\psi(x) = E\psi(x)$ give the meaning of the symbols in this equation. [4 marks]

f) Given that:- $J_x = \left(\frac{\hbar}{2mi}\right)\left[\psi^* \frac{d\psi}{dx} - \left(\frac{d\psi}{dx}\right)\psi^*\right]$, show that $J_x = \left(\frac{\hbar k}{m}\right)(A^2 - B^2)$ if $\psi = Ae^{ikx} + Be^{-ikx}$. [6 marks]

g) Consider a one-dimensional particle which is confined in a region $0 < x < a$ whose wave function is $\psi(x,t) = sin\left(\frac{\pi x}{a}\right) exp(-i\omega t)$. Find the potential energy $V(x)$ of the region. [4 marks]

h) Show that $i[\hat{A}, \hat{B}]$ will be Hermitian if $\hat{A}$ and $\hat{B}$ are Hermitian operators. [3 marks]

QUESTION TWO [20 MARKS]

a) What boundary conditions do wave functions obey? [2 marks]

(b) A particle confined to a one dimensional potential well has a wave function given by:-

$$\psi(x) = \begin{cases} 0 & for \; x < -L/2 \\ Acos\left(\frac{3\pi x}{L}\right) & for \; -L/2 \le x \le L/2 \\ 0 & for \; x > L/2 \end{cases}$$

(i) Sketch the wave function $\psi(x) \approx cos\left(\frac{3\pi x}{L}\right)$ [3 marks]

(ii) Calculate the normalization constant A [5 marks]

(iii) Calculate the probability of finding the particle in the interval $-L/4 \leq x \leq L/4$ [5 marks]

(iv) Using the Schrödinger equation $\left(-\frac{\hbar^2}{2m}\right)\left(\frac{d^2\psi}{dx^2}\right) = E\psi$ show that the energy E corresponding to this wave function is given by:- $\frac{9\pi^2\hbar^2}{2mL^2}$. [5 marks]

QUESTION THREE [20 MARKS]

A particle moving in one dimension is in a stationary state whose wave function is given by:-

$$\psi(x) = \begin{cases} 0 & for\ x > -a \\ A\left(1 + cos\frac{\pi x}{a}\right) & for - a \leq x \leq a \\ 0 & for\ x > a \end{cases}$$

a) Find the magnitude of A so that $\psi(x)$ is normalized. [5 marks]

b) Evaluate Δx and Δp hence verify that $\Delta x \Delta p \geq \frac{\hbar}{2}$. [15 marks]

QUESTION FOUR [20 MARKS]

A particle of mass m, moves freely inside an infinite potential well of length a, has an initial wave function given by:-

$\psi(x, 0) = \frac{A}{\sqrt{a}} sin\left(\frac{\pi x}{a}\right) + \sqrt{\frac{3}{5a}} sin\left(\frac{3\pi x}{a}\right) + \frac{1}{\sqrt{5a}} sin\left(\frac{5\pi x}{a}\right)$ where A and a are real constants, a=2 and $\phi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$.

a) Find A so that $\psi(x, 0)$ is normalized [4 marks]

b) What are the values of measurements of energy and their corresponding probabilities? [7 marks]

c) Calculate the average energy. [3 marks]

d) Find the wave function $\psi(x, t)$ at any later time t. [3 marks]

e) Determine the probability of finding the system at any later time t in state $\varphi(x, t) = \sqrt{\frac{2}{a}} sin\left(\frac{5\pi x}{a}\right) e^{-iE_5 t/\hbar}$. [3 marks]

QUESTION FIVE [20 MARKS]

a) The wave function of a hydrogen atom is given by:- $\psi = \frac{1}{\sqrt{\pi a^3}} e^{-r/a}$, find $\langle r \rangle$ and $\langle r^2 \rangle$ for an electron in the ground state express your answer in terms of Bohr's radius (a) [6 marks]

b) Find $\langle x \rangle$ and $\langle x^2 \rangle$ for an electron in the ground state of a hydrogen atom if $r^2 = x^2 + y^2 + z^2$. [2 marks]

20

c) Find $\langle x^2 \rangle$ in the state $n = 2, l = 1$ and $m = 1$ if $x = r \sin\theta \cos\phi$ [6 marks]

d) Find the probability that an electron in the ground state of a hydrogen atom will be found inside the nucleus? [6 marks]