(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS

COURSE CODE: **MAP 322**

COURSE TITLE: **GROUP THEORY II**

DATE: 29/08/2022 **TIME:** 2:00 PM - 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question ONE and Any TWO Questions

TIME: 2 Hours

QUESTION ONE (30MARKS)

a. Define the following
   i. Center of a Group (2marks)
   ii. P-groups (2marks)
   iii. Normalizer (2marks)

b. State the following
   i. First Sylow Theorem (2 marks)
   ii. Second Sylow Theorem (2 marks)

c. Determine
   i. The center of D_4 (2marks)
   ii. Conjucacy classes of D_4 (3marks)

d. Let G be a group of order p^n where p is prime. Show that G has a non-trivial center (10 marks)

e. Determine the order of A_5. Hence the order of its subgroups (5 marks)

QUESTION TWO (20MARKS)

a. Define the following sets
   i. Chief series (2marks)
   ii. Soluble group (2marks)

b. Show that all finite abelian groups are soluble (8marks)

a. Show that every finite group G has a composition series (8marks)

QUESTION THREE (20MARKS)

a. Define the following
   i. Nilpotent group (2marks)
   ii. Central series (2marks)

b. Show that If G is a finite group and P is a Sylow p-subgroup of G then $N_G(N_G(P)) = N_G(P)$ (10marks)

c. Show that a group G is nilpotent if and only if it has a central series (6marks)

QUESTION FOUR (20MARKS)

a. State the following theorems

   i. The Fundamental Theorem of Finitely Generated Abelian Groups (3marks)

   ii. Fundamental Theorem of Finite Abelian Groups (3marks)

b. Show that a finite abelian group is a p-group if and only if its order is a power of p. (7marks)

c. Classify all abelian groups of order $540 = 2^2.3^3.5$ using the fundamental theorem of finite abelian groups (7marks)

QUESTION FIVE (20MARKS)

a. Define the following

   i. External direct product (2marks)

   ii. Internal direct product (2marks)

b. Show that if G is the internal direct product of H and K, then G is isomorphic to the external direct product H×K (8marks)

c. Let H and K be groups and let $\rho: K \longrightarrow Aut(H)$ be a group homomorphism. Show that the binary operation $(H \times K) \times (H \times K) \longrightarrow (H \times K)$ endows H×K with a group structure with identity element (1,1) (8marks).