



*(Knowledge for Development)*

**KIBABII UNIVERSITY**  
**UNIVERSITY EXAMINATIONS**  
**2021/2022 ACADEMIC YEAR**  
**FIRST YEAR FIRST SEMESTER**  
**SPECIAL/SUPPLEMENTARY EXAMINATION**  
**FOR THE DEGREE OF MASTER OF SCIENCE IN PURE AND**  
**APPLIED MATHEMATICS**

**COURSE CODE:** MAT 817/869  
**COURSE TITLE:** COMPLEX ANALYSIS I  
**DATE:** 15/07/2022 **TIME:** 8 AM -11 AM

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**INSTRUCTIONS TO CANDIDATES**

Answer Any THREE Questions

TIME: 3 Hours

This Paper Consists of 2 Printed Pages. Please Turn Over.

**QUESTION ONE [20 MARKS]**

- (a) Find the Laurent series about the indicated singularity for the function

$$f(z) = \frac{1}{(z+1)(z+3)} \quad z > 3 \quad (4 \text{ mks})$$

- (b) Evaluate
- $\oint_C \frac{2z^2+z}{z^2-1} dz$
- where C is a circle
- $|z-1|=1$
- (5 mks)

- (c) Evaluate
- $(3x+y)dx + (2y-x)dy$

(i) Along the curve  $y = x^2 + 1$  (5 mks)

(ii) Straight lines from (0,1) to (0,5) and then (0,5) to (2,5) (6 mks)

**QUESTION TWO [20 MARKS]**

(a) Show that  $\sec^{-1}(z) = \frac{1}{i} \ln \left( \frac{1+\sqrt{1-z^2}}{z} \right)$  (5 mks)

(b) Find the residuals of the function  $f(z) = \frac{z^2}{(z-1)^2(z+2)}$  (5 mks)

(c) Evaluate  $\oint_C (7x-2y+7)dx + (3x-4y-10)dy$  around a triangle in the xy plane with vertices at (0,0), (2,0) and (2,3) (5 mks)

(d) Determine the number of zeros of  $z^6 + 5z^2 - z = 1$  interior to  $|z|=1$  (5 mks)

**QUESTION THREE [20 MARKS]**

(a) Prove that the function  $f_1(z) = \int_0^\infty t^3 e^{-zt} dt$  is analytic at all points of z for which  $\text{Re}z > 0$  (6 mks)

(b) State and prove the Rouché's theorem (14 mks)

**QUESTION FOUR [20 MARKS]**

- (a) State the following terms

(i) Analytic continuation (2 mks)

(ii) Conformal mapping (2 mks)

- (b) If
- $f(z) = z^5 - 2z^3 + 3z + 2 - i$
- , evaluate

$$\int_C \frac{f'(z)}{f(z)} dz$$
 where C encloses all zeros of  $f(z)$  (4 mks)

(c) Determine the linear fractional transformation that maps  $z = 0, -i, -1$  onto  $w = i, 1, 0$  respectively (6 mks)

(d) Evaluate  $\oint_C (z - \text{Re}(z)) dz$   $C: |z|=2$  (6 mks)

**QUESTION FIVE [20 MARKS]**Consider the triangle  $P(0,0)$ ,  $Q(2,0)$  and  $R(2,4)$ 

(i) Draw the triangle and its image under  $T(z) = 2z^2 + (3-2i)$  (12 mks)

(ii) Discuss conformity of  $T$  at  $P(0,0)$  and  $R(2,4)$  (8 mks)