KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS
2020/2021 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER
SPECIAL/SUPPLEMENTARY EXAMINATIONS

FOR THE DEGREE OF BACHELOR OF SCIENCE (PHYSICS)

COURSE CODE: SPC 321

COURSE TITLE: QUANTUM MECHANICS I

DATE: 10/1/2022 **TIME:** 11-1PM

INSTRUCTIONS TO CANDIDATES

TIME: 2 HOURS

Answer question ONE and any TWO of the remaining

QUESTION ONE [30 MARKS]

a) Show that $[\hat{x}, \hat{p}_x] = i\hbar$ [3 marks]

b) An Eigen function of the operator $\frac{d^2}{dx^2}$ is $\psi = e^{2x}$. Find the corresponding Eigen value [3 marks]

c) Find the probability that a particle trapped in a box L wide can be found between o.45L and 0.55L for ground and first excited states [4 marks]

d) A particle has a 1-dimensional wave function given by:- [3 marks]
$\psi(x) = \begin{cases} ax, & 0 \leq x \leq 1 \\ 0, elsewhere \end{cases}$. Find the expectation value of x.

e) The Hamiltonian of a simple harmonic oscillator is given by $\widehat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega\hat{x}^2$ [4 marks]
prove that: $[\hat{x}, \widehat{H}] = \frac{i\hbar p_x}{m}$.

f) State two conditions satisfied by a well behaved wave function. [2 marks]

g) Verify that spherical harmonics ψ_{11} and $\psi_{21}$are orthogonal. [4 marks]

h) At t=0 a particle is represented by a 1-dimensional wave function given by:- [5 marks]
$\psi(x, 0) = \begin{cases} Ax/a\,, if\ 0 \leq x \leq a \\ A(b-x)/(b-a), if\ a \leq x \leq b \\ 0, otherwise \end{cases}$. Normalize ψ and sketch $\psi(x, 0)$ as a function of x.

i) What is the parity of 1s and 2p states of a hydrogen atom? [2 marks]

QUESTION TWO [20 MARKS]

a) A steady stream of particles with energy $E > V_0$ is incident on a potential step of height V_0. The wave functions in the two regions are given by:
$\psi_1(x) \exp(ik_1x) + A\exp(-ik_1x)$ and $\psi_2(x) = B\exp(ik_2x)$

(i) Write down expressions for quantities k_1 and k_2 in terms of E and V_0 [2 marks]

(ii) Show that: $A = \left[\frac{k_1-k_2}{k_1+k_2}\right]A_0$ and $B = \left[\frac{2k_1}{k_1+k_2}\right]A_0$ [6 marks]

(iii) Determine the reflection and transmission coefficients in terms of quantities k_1 and k_2. [4 marks]

(iv) If $E = \frac{4V_0}{3}$ calculate the values of reflection and transmission coefficients. [4 marks]

b) Show that for a simple harmonic oscillator in ground state the probability for finding particles in the classical forbidden region is approximately 16%. [4 marks]

QUESTION THREE [20 MARKS]

a) Show that the momentum operator $-i\hbar\frac{\partial}{\partial x}$ is a Hermitian operator. Hence obtain Eigen function and Eigen values of $\hat{p}_x$. [10 marks]

b) A particle of mass m and energy E is trapped in a square well of depth 2a and depth $V_0 > E$. Find the normalized wave functions of the particle and hence sketch the first two wave functions in the three regions. [10 marks]

QUESTION FOUR [20 MARKS]

a) The spin wave functions of two electrons is: $\frac{(x\uparrow x\downarrow - x\downarrow x\uparrow)}{\sqrt{2}}$. What is the Eigen value of $S_1.S_2$? [5 marks]

b) Show that for a neutron-proton system; $\sigma_p\sigma_n = \begin{cases} -3, & for\ singlet\ state \\ 1, & for\ triplet\ state \end{cases}$ [7 marks]

c) Given that: $L = rxp$, show that $[L_x, L_y] = i\hbar L_z$. [8 marks]

QUESTION FIVE [20 MARKS]

a) Find the expectation values of kinetic energy, potential energy and total energy of hydrogen atom in ground state for $\psi_0 = \frac{e^{-r/a_0}}{\sqrt{\pi a_0^3}}$ where a_0 is the Bohr's radius. [8 marks]

b) Consider a three-dimensional spherically symmetrical system whose Schrödinger's equation can be decomposed into radial equation and angular equation. The angular equation is given by: [12 marks]
$\lambda\psi(\theta,\varphi) = -\left[\left(\frac{1}{sin\theta}\right)\left(\frac{\partial}{\partial\theta}\right)\left(sin\theta\frac{\partial}{\partial\theta}\right) + \left(\frac{1}{sin^2\theta}\right)\left(\frac{\partial^2}{\partial\varphi^2}\right)\right]\psi(\theta,\varphi)$. solve the equation and hence derive the quantum number *m*.