(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

SPECIAL/SUPPLIMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: MAT 401

COURSE TITLE: TOPOLOGY I

DATE: 10/01/2022 **TIME:** 8:00 - 10:00 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30marks)

a). Define the following terms

i). An $\epsilon - ball$ in a topological space (2 Marks)

ii). A topological space (3 Marks)

iii). A continuous function in a metric space (2 Marks)

iv). A quotient topology (2 Marks)

b). Prove that every finite set in a Hausdorff space X is closed (5 Marks)

c). Define a cluster point hence show that any $a \in [0,1]$ is a cluster point. (5 Marks)

d). Prove that if $f: X \to Y$ is a continuous function between two topological spaces X and Y then for every $A \in X$, $f(\bar{A}) \subseteq \overline{f(A)}$. (6 Marks)

e). Define a homeomorphism hence show that any linear function is a homeomorphism from $\mathbb{R}$ the usually topological space on $\mathbb{R}$ to itself. (5 Marks)

QUESTION TWO (20marks)

a). Let X be a metrizable topological space and $A \subset X$. Prove that if there is a sequence of points of A converging to x then $x \in \bar{A}$ $(closure\ of\ A)$ and the converse is true. (8 marks)

b). Consider the set $Y = (0,3) \cup (4,8]$

(i). Find the closure of $(0,3)$ in Y and in $\mathbb{R}$. (4 marks)

(ii). Find the interior of $(4,8]$ (2 marks)

(iii). Show that $(0,3)$ is both closed and open. (6 marks)

QUESTION THREE (20marks)

a). (i). Let $X = \{a, b, c\}$ form two topologies τ_1 and τ_2 such that $\tau_1 \subset \tau_2$. (4 Marks)

(ii). Prove that indeed τ_2 is a topology. (4 Marks)

c). Let X be a topological space. Prove that the following conditions holds

i). ϕ and X are closed (2 Marks)

ii). Arbitrary intersection of closed sets are closed (5 Marks)

iii). Finite union of closed sets are closed (5 Marks)

QUESTION FOUR (20marks)

a). Let (X, τ) be a topological spaces and $Y \subseteq X$ such that $\tau_Y = \{Y \cap U : U \in \tau\}$. Prove that τ_Y is a topology on Y hence, describe the basis of that topology with respect to that of τ. (10 Marks)

b). Consider the set $Y = \{e, f, g\}$ and a topology on X defined by $\tau_X = \{\phi, \{f\}, \{g\}, \{e\}, \{e, f\}, \{g, f\}, \{g, e\}, Y\}$.

(i). Find all neighborhoods of points e and g in Y. (4 marks)

(iii). Find all the cluster points of the set $A = \{g\}$. (4 marks)

c). What do you understand by the usual topology on $\mathbb{R}$. (2 marks)

QUESTION FIVE (20marks)

a). Let (X, τ_f) a topological space where $\tau_f = \{U : U \subseteq X \mid X - U \; is \; finite \; . \text{ or all of } X\}$ Show that τ_f is a topology. (7 Marks)

b). Let X and Y be topological spaces and $f: X \to Y$ a function. Show that if for every closed set $B \in Y$ the set $f^{-1}(B)$ is closed in X then f is continuous. (5 Marks)

c). Define a Hausdorff space hence prove that any convergent sequence in a Hausdorff space has at most one limit (8 Marks)