(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

SPECIAL/SUPPLIMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: **MAT 403**

COURSE TITLE: **COMPLEX ANALYSIS II**

DATE: 11/01/2022 **TIME:** 2:00 – 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (Compulsory)(30 marks)

a) Show that $\int_{-\infty}^{\infty} \frac{z^2+3}{(z^2+1)(z^2+4)} dz = \frac{5}{6}\pi$ (10 marks)

b) Determine the singularities of the following functions hence find their Laurent series

i) $f(z) = (z-3)\sin\frac{1}{z+2}$ (5 marks)

ii) $f(z) = \frac{e^{2z}}{(z-1)^3}$ (5 marks)

c) Define a Harmonic function and hence show that the function $\emptyset = x^3 - 3xy^2 + 2y$ can be a real part of analytic function. Find the imaginary part of the analytic function.

(10 marks)

QUESTION TWO (20 marks)

a) State and prove the Residue theorem. (5 marks)

b) Show that $\oint_c \frac{\sin z}{z^4} dz = -\frac{\pi}{3} i$, where $c: \ |z| = 1$,described in a positive direction.

(5 marks)

c) Let $f(z)$ be analytic inside and on a simple closed curve C except at a pole a of order m inside C . Prove that the residue of $f(z)$ at a is given by

$a_{-1} = \lim_{z\to a} \frac{1}{(m-1)!} \frac{d^{m-1}}{dz^{m-1}}\{(z-a)^m f(z)\}$. Hence find the residue of the function

$f(z) = \frac{z^2-2z}{(z+1)^2(z^2+4)}$. (10 marks)

QUESTION THREE (20 marks)

a) Find $I = \int_0^{2\pi} \frac{\cos 2\theta d\theta}{5-4\sin\theta}$ (10 marks)

b) Compute the integral $\oint \frac{5z-2}{z(z-2)} dz$ around a circle radius $r = 3$ centered at the origin.

(5 marks)

c) Discuss the singularity of the following function: $f(z) = \frac{z \cos z}{(z-1)(z^2+1)^2(z^2+3z+2)}$

(5 marks)

QUESTION FOUR (20 marks)

a) Evaluate $\int_{-\infty}^{\infty} \frac{z^2 dz}{(z^2+1)^2(z^2+2z+2)}$ (5 marks)

b) Show that $\int_0^{2\pi} \frac{d\theta}{3-2\cos\theta+\sin\theta} = \pi$ (5 marks)

c) Consider the contour C defined by $x = y, x > 0$ and the contour C_1 defined by $x = 1, y \geq 1$. Maps these two curves using $w = \frac{1}{z}$ and verify that their angle of intersection is preserved in size and direction. (10 marks)

QUESTION FIVE (20 marks)

a) Find a Schwartz-Christoffel transformation that maps the upper half plane H to the inside of a triangle vertices $-1, 0$ and i. (10 marks)

b) Define the Laurent series of a function of a complex variable $f(z)$ and hence expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent series valid for $1 < |z| < 3$ (10 marks)