(15)

(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **MAA 222/MAT 322**

COURSE TITLE: **OPERATION RESEARCH**

DATE: 26/07/2022 **TIME:** 1:00 PM - 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE

a) Define Model, discuss the steps of Modelling

Solve the following LP problem using Simplex Method.

Maximize $z = 6x_1 + 8x_2$

Subject to

$$5x_1 + 10x_2 \leq 60$$

$$4x_1 + 4x_2 \leq 40$$

$$x_1 \quad and \quad x_2 \geq 0$$

(5 Marks)

Hence find the new solutions if

(i) The Right Hand side constants of the constraint 1 and constraint 2 are changed from 60 and 40 to 40 and 20 respectively.

(ii) The Right Hand side constants of the constraint 1 and constraint 2 are changed from 60 and 40 to 20 and 40 respectively.

(iii) Determine the range of optimality

(iv) If a new constraint $6x_1 + 3x_2 \leq 48$ is added. (15marks)

QUESTION TWO (20mks)

(a) What are types of transportation problem? Explain them with suitable examples. (6 Marks)

(b) A dairy farm has three plants located throughout a city. Daily mild production at each plant is as follows.

Plant 1 - 6 million litres

Plant 2 - 1 million litres

Plat 3 - 10 million litres

Each day the farm must fulfil the needs of four distribution centres. Minimum requirement at each centre is as follows.

Distribution centre 1 - 7 million litres.

Distribution centre 2 - 5 million litres.

Distribution centre 3 - 3 million litres.

Distribution centre 4 - 2 million litres.

The cost of shipping one million litres of milk form each plant to each distribution center is given in the following table in hundreds of shillings.

Plants	Distribution centres			
	1	2	3	4
1	2	3	11	7
2	1	0	6	1
3	5	8	15	9

The dairy farm wishes to decide as to how much should be the shipment from which plant to which plant to which distribution centre so that the cost of shipment may be minimum.

(v) Formulate the transportation matrix
(vi) Obtain the initial feasible solution using the following methods. Northwest corner cell method, Least cost cell method and Vogel's Approximation Method.
(vii) Find the optimal solution (14 Marks)

QUESTION THREE (20 MKS)

a) Form the dual of the following LP problem.

Maximize $z = 5x_1 + 6x_2$

Subject to

$$4x_1 + 7x_2 \leq 20$$
$$5x_1 + 2x_2 \leq 10$$
$$6x_1 + 8x_2 \leq 25$$

x_1 *and* x_2 are unrestricted in sign. (3 Marks)

b) A college is having a degree programme for which the effective semester time available is very less and the programme requires fieldwork. Hence a few hours can be saved from total number of class hours and can be utilized for the fieldwork. Based on past experience, the college has estimated the number of hours required to teach each subject by each faculty. The course in its present semester has 5-subjects and the college has considered 6 existing faculty members to teach these courses.

The objective is to assign the best 5 teachers out of these 6 faculty members to teach 5 teach 5 different subjects so that the total number of class hours required is minimized. The data is given in the table below.

Solve this assignment problem optimally using the Hungerian Method.

Faculty	Subject 1	2	3	4	5
1	30	39	31	38	40
2	43	37	32	35	38
3	34	41	33	41	34
4	39	36	43	32	36
5	32	49	35	40	37
6	36	42	35	44	42

(9 Marks)

(c) Explain the similarities and differences between transportation problems and assignment problems

QUESTION FOUR (20MKS)

a) Solve the following Linear programming problem using the Big M method.
Minimize $z = 24x_1 + 30x_2$

Subject to: $2x_1 + 3x_2 \geq 10$

$4x_1 + 9x_2 \geq 15$

$6x_1 + 6x_2 \geq 20$

$x_1 \quad and \quad x_2 \geq 0$

(8Marks)

b) In a multi-speciality hospital, nurses report to duty at the end of every four hours as shown in a table below. Each nurse, after reporting, will work for 8 hours continuously. The minimum number of nurses required during various periods are summarized in the table below. Develop a Mathematical Model to determine the number of nurses to report at the beginning of each period such that the total number of nurses who have to report to duty in a day is minimized.

Internal number	Time Period		Minimum number of nurses required
	From	To	
1	12 midnight	4.00a.m	20
2	4.00a.m.	8.00a.m	25
3	8.00a.m	12Noon	35
4	12 Noon	4.00p.m	32
5	4.00p.m	8.00p.m	22
6	8.00p.m.	12midnight	15

(4 Marks)

c) Consider the following Linear Programming Model and solve it using the two-phase Method

Minimize $z = 12x_1 + 18x_2 + 15x_3$

Subject to: $4x_1 + 8x_2 + 6x_3 \geq 64$

$3x_1 + 9x_2 \geq 15$

$6x_1 + 6x_2 + 12x_3 \geq 96$

$x_1 \quad x_2 \quad and \quad x_3 \geq 0$

(8 Marks)

QUESTION FIVE (20 MKS)

A farmer requires to feed his pigs as cheaply as possible. The pigs requires the diet consisting of a minimum amount of three nutrients N_1,N_2 and N_3 which form a part of the commercially available food stuffs F_1, F_2, F_3 and F_4.The number of units contained in each food stuff, cost per unit of each and minimum requirements are given below

Nutrients	Food stuffs				Minimum Requirements
	F_1	F_2	F_3	F_4	
N_1	5	8	4	1	50
N_1	3	8	7	5	40
N_3	4	0	5	4	8
Cost/unit	1.0	0.9	1.2	0.9	

(I) set up a linear programming model for this problem

(II) hence solve it using the dual simplex method