(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER

SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF MASTER OF SCIENCE IN PURE MATHEMATICS

COURSE CODE: **MAT 811**

COURSE TITLE: **ABSTRACT INTEGRATION I**

DATE: 25/07/2022 **TIME:** 8:00 AM - 10:00 AM

Answer Any other THREE Questions

TIME: 2 Hours

QUESTION ONE (20 MARKS)

a) Define the following terms

i. Ring

ii. σ –Ring

iii. Algebra

iv. $\sigma -$ Algebra

v. Family

b) Show that if $f: X \rightarrow Y$ and ρ is a σ –Ring of subsets of X, then the class of all sets $B \subset Y$ such that $f^{-1}(B)$ is in ρ , is a σ –Ring of subsets of Y

c) Show that if R is a ring of subsets of a set X, then $M(R) = G(R)$

QUESTION TWO (20 MARKS)

a) State the Lemma on Monotone classes (LCM)

b) Define the following terms

i. Set function

ii. Additive

iii. Measure

iv. Continuous from below

c) Show that if R is a ring and μ is an extended real valued set function on R which is positive, countably additive and satisfies the condition $\mu(\emptyset) = 0$, then μ is a measure

QUESTION THREE (20 MARKS)

a) Show that if v is an outer measure, the class M of v-measurable sets is a ring

b) Let R be a ring, and suppose that μ_1 *and* μ_2 are measures on the $\sigma -$ Ring $G(R)$ generated by R such that $\mu_1(E) = \mu_2(E)$ for all E in R. Assume moreover, that the restriction of μ_1 to R is σ –finite, then show that $\mu_1 = \mu_2$ on $G(R)$

QUESTION FOUR (20 MARKS)

a) Define the following terms

i. Measurable space

ii. Locally measurable

iii. Measurable function f

iv. Characteristic function

v. Borel measurable

a) Show that if $f: X \to \mathbb{R}$ is a function such that $N(f)$ is measurable, then each of the following conditions is necessary and sufficient for the measurability of f :

   i. $\{x: f(x) < c\}$ is locally measurable for each real number c
   ii. $\{x: f(x) \leq c\}$ is locally measurable for each real number c
   iii. $\{x: f(x) > c\}$ is locally measurable for each real number c
   iv. $\{x: f(x) \geq c\}$ is locally measurable for each real number c

b) Show that if $f \text{ and } g$ are measurable, then $f + g$ is also measurable

QUESTION FIVE (20 MARKS)

a) Show that if f and g are simple functions, c is a real number and A is a locally measurable set, then all of the following are simple

   i. cf
   ii. $f + g$
   iii. $|f|$
   iv. $f \cup g$
   v. $f \cap g$
   vi. f^+, f^-
   vii. $\chi_A f$
   viii. fg

b) Show that if f is a measurable function, c is a real number and $c > 0$, then $f \cap c$ is a measurable function

c) Show that if f is a measurable function, then there exists a sequence of simple functions f_n such that f_n converges to f pointwise on X, that is, $f_n(x) \to f(x)$ for each x in X. If moreover, $f \geq 0$ one can make $0 \leq f_n \uparrow f$