(16)

(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

SECOND YEAR SUPPLEMENTARY

EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: MAT 252

COURSE TITLE: ENGINEERING MATHEMATICS II

DATE: WED 27/07/2022 TIME: 2:00 PM – 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS): COMPULSORY

a) A particle moves so that its position vector is given by $\vec{r} = 2\cos\omega t\, i + 2\sin\omega t\, j$, where ω is a constant. Show that the velocity $\vec{v}$ of the particle is perpendicular to $\vec{r}$. (5 marks)

b) If $B = 3z^2 + 4i$, find Laplacian of B (5 marks)

c) If $\phi(x, y, z) = 3x^2 y - y^3 z^2$, find $\nabla\phi$ at the point $(1,-2,-1)$. (5 marks)

d) Show that for the complex variable z the following formula is valid: $\sin 2y = 2\sin y \cos y$ (5 marks)

e) Find $\nabla\phi$ if $\phi = \frac{1}{r}$. (5 marks)

f) Find the work done in moving a body along a straight line from $(5,3,-1)$ to $(3,-2,2)$ in a force field given by $\vec{F} = 2i - j + 4k$. (5 marks)

QUESTION TWO (20 MARKS)

a) Classify according to type and determine the characteristics of the following p.d.e: $2u_{xx} - 4u_{xy} - 6u_{yy} + u_x = 0$ (6 marks)

b) Calculate e^z when $z = 1 + \frac{\pi}{4}i$ (5 marks)

c) If $\vec{F} = (2xy + z^3)i + x^2 j + 3xz^2 k$

(i) Show that it is a conservative force field (5 marks)

(ii) Find the work done in moving an object in this field from $(1,-2,1)$ to $(3,-1,4)$ (4 marks)

QUESTION THREE (20 MARKS)

a) If $U = a + ib$ and $V = c + id$, prove that $\overline{UV} = \overline{U} \times \overline{V}$ (6 marks)

b) Prove that $u = e^{-x}(x\sin y - y\cos y)$ is harmonic (5 marks)

c) A particle moves along a curve whose parametric equations are $x = e^{-t}, y = 3\cos 2t, z = 3\sin 2t$, where t is time. Determine:

(i) Velocity at time t (5 marks)

(ii) Acceleration at time t (4 marks)

QUESTION FOUR (20 MARKS)

a) Show that the function $f(x) = x^2 - y^2 - 2ixy$ is analytic in the entire complex plane (8 marks)

b) Given that: $\phi = 2x^3y^2z^4$, find $\nabla \bullet \nabla\phi$ (6 marks)

c) Find the following: $2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) \bullet 3\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$ (6 marks)

QUESTION FIVE (20 MARKS)

a) A scalar field $v = xyz$ exists over a curved surface defined by $x^2 + y^2 = 4$ between the planes $z = 0$ and $z = 3$ in the first octant. Evaluate $\int_s v d\vec{s}$ over this surface. (6 marks)

b) Find the Fourier series representing $f(x) = x : 0 \le x \le 2\pi$ (8 marks)

c) If $\vec{A} = xz^3\,i - 2x^2yz\,j + 2yz^4\,k$, find $\nabla \times \vec{A}$ at the point $(1,-1,1)$. (6 marks)

END