(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: MAT 222/MAA 221

COURSE TITLE: CALCULUS III

DATE: 27/07/2022 **TIME:** 2 PM -4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) Define the following terms

(i) Local minimum (2 mks)

(ii) Local maximum (2 mks)

b) Evaluate $\lim\limits_{(x,y)\to(0,0)} \frac{x^2-xy}{\sqrt{x}-\sqrt{y}}$ (4 mks)

c) Suppose x is a differentiable function near each (x, y) for the equation $zsiny + x^2z - 2e^{xyz} = 11$ find $\frac{\partial z}{\partial y}$ (3mks)

d) The production function is given by $f(x, y) = 4xy$ maximize the this function subject to budget constraint $x - 2y = 10$ (5 mks)

e) Find the radius and interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(-2)^n(x-3)^n}{n5^{n-1}}$ (5 mks)

f) Find the volume in the 1st octant between the planes $z = 0$, and $z = 2x - y + 3$ And inside the cylinder $x^2 + y^2 = 9$ (5 mks)

g) Verify that the Tailor series expansion for the function $f(x) = sinx$ about $x = 0$ is $sinx = \sum_{n=0}^{\infty} \frac{(-1)^n(x)^{2n+1}}{(2n+1)!}$ hence find the Maclaurin series for $f(x) = 2xsinx$ (6 mks)

QUESTION TWO (20 MARKS)

a) For what values does the series converge $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n}$ (4 mks)

b) Consider the series $\sum_{k=1}^{\infty} \frac{(-3)^{k-1}2^k}{3k^2}$ use ratio theorem to show that the series diverges (4 mks)

c) Find the area of the portion of the cone $x^2 + y^2 = 2z^2$ lying above the xy-plane and inside the cylinder $x^2 + y^2 = 4y$ (6 mks)

d) An open cylinder has a surface area of $162.34\ cm^2$ Find the radius and the height that will yield minimum volume (6 mks)

QUESTION THREE (20 MARKS)

a) Use the 1st principles to determine $\frac{\partial f}{\partial y}$ given that $f(x,y) = 2xy - y^2 - 3xy^3$ (4 mks)

b) Let $f(x,y,z) = xln(xy) - e^{x^2z} + 3\cos(\text{xyz})$. Find

(i) f_{xx} (2 mks)

(ii) f_{xyz} (3 mks)

c) Evaluate $\int_0^{\pi}\int_0^{\frac{\pi}{4}}\int_0^{siny} 4sinzcosydxdydz$ (5 mks)

d) Locate and classify all critical points of

$f(x_1, x_2) = 3x_1^2 - x_2^3 - 6xy$ (6 mks)

QUESTION FOUR (20 MARKS)

a) Investigate the convergence of $\sum_{k=0}^{\infty}\frac{(-3)^k e^k}{k!}$ (6 mks)

b) Use the Lagrange multipliers to find the local extrema of the function

$f(x,y) = y^2 - 4x$ Subject to $x^2 + y^2 = 25$ (7 mks)

c) Locate any relative extreme points and determine their nature for the function

$f(x,y,z) = -2x^3 + 6xz + 2y - y^2 - 6z^2 + 3$ (7 mks)

QUESTION FIVE (20 MARKS)

a) Let $z = e^{3x}siny$ and $x = s^2t^2 - 2t$ and $y = s^3 - 4t$ find

$\frac{\partial z}{\partial t}$ and $\frac{\partial z}{\partial s}$ (7 mks)

b) If $R = \{x, y \ / \ 1 \le x \le 2 \ and \ 0 \le y \le 3\}$ evaluate $\iint_R (-4xy^2 - 2x^3y + 3)dA$ (3 mks)

c) Find the volume of the solid bounded by the graphs of $z = 4 - y^2$,

$x - z = 2, x = 0$, and $z = 0$ (5 mks)

d) Consider the series $S_n = \frac{1}{\sqrt{2n-1}}$ using the integral test,

determine whether the series converges or diverges (5 mks)