(18)

(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

END OF SEMESTER EXAMINATIONS

THIRD YEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **MAA 314**

COURSE TITLE: **METHODS I**

DATE: 13/01/2022 **TIME:** 2:00 – 4:00 PM

INSTRUCTIONS

Answer **Questions ONE and** Any other **TWO**

QUESTION ONE [30MKS]

a. Show that $\Gamma(x+1)=\Gamma(x)$ (3mks)

b. Determine $L^{-1}\left\{\frac{4s^2-5s+6}{(s+1)(s^2+4)}\right\}$ (5mks)

c. Obtain half range cosine series to represent the function $f(x)=2x$ defined over $0<x<\pi \quad for \quad f(x+2\pi)=f(x)$ (6mks)

d. Show that if $z=\frac{x}{y}\ln y$, then $\frac{\partial z}{\partial y}=x\frac{\partial^2 z}{\partial y \partial x}$ (3mks)

e. Use special functions to evaluate (4mks)

$$\int_0^1 x^4(1-x^2)^{\frac{1}{2}}dx$$

f. Find the singular point for $2x^2y''-xy'+(1-x)y=0$ and show if the equation can be solved using Frobenius method. (4mks)

g. Find the 1st three nonzero terms for the Legendre series

$$f(x)=\begin{cases}0; & -1\le x\le 1\\ 1; & 0\le x\le 1\end{cases}$$ (5mks)

QUESTION TWO [20MKS]

a. Evaluate $\int_0^1 \ln\left(\frac{1}{x}\right)^{\frac{3}{2}}dx$ (4mks)

b. Prove that $B(m,\ n)=\frac{\Gamma m \Gamma n}{\Gamma(m+n)}$ (6mks)

c. solve the Legendre's differential equation $(1-x^2)y''-2xy'+6y=0$ (10mks)

QUESTION THREE

a. Transform the following differential equation into a Legendre equation by letting $x=\cos\theta$ (7mks)

b. Derive the duplication formula and use it to find $\Gamma(3\frac{1}{2})$ (6mks)

c. Find the solution of $\frac{\partial U}{\partial x}+\frac{\partial U}{\partial y}=0$ (7mks)

QUESTION FOUR [20MKS]

a. Evaluate $\int_0^\infty \sqrt{z}\, e^{-z^3}dz$ (3mks)

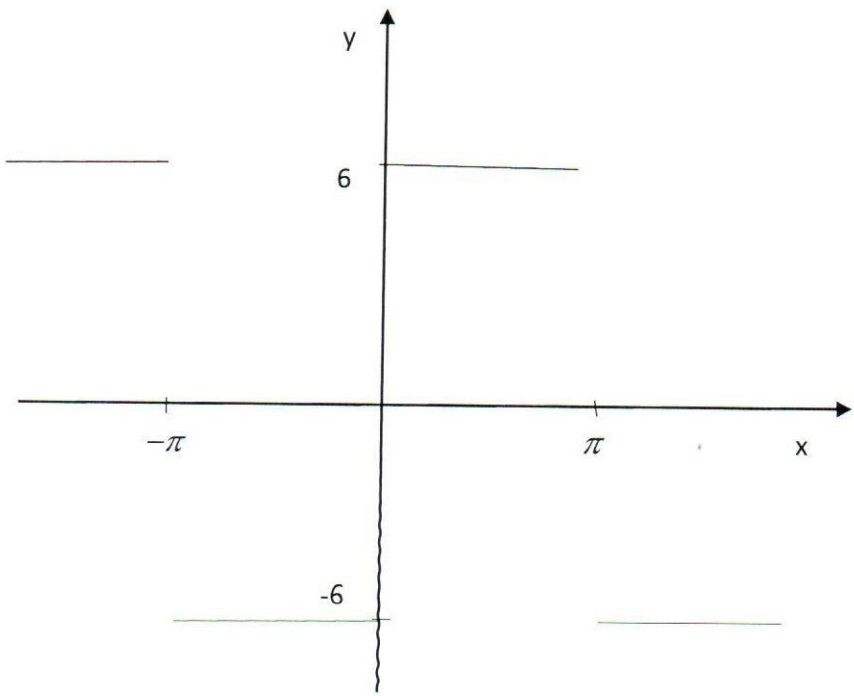
b. Use Laplace transform to solve (7mks)

i. $3\dot{x} - 4x = \sin 2t \;\; at \;\; t = 0, \;\; x = \frac{1}{3}$

c. Find the general solution of the equation

$9x^2 y'' + 9xy' + (9x^2 - \frac{1}{4})y = 0$ (5mks)

d. Find the Fourier series for the given function. (5mks)



QUESTION FIVE [20MKS]

a. Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$ (5mks)

b. Expand the following functions in Legendre series $f(x) = P_3'(x)$ (5mks)

c. Sol Solve the differential equation following using the power series (10mks)

$y' - 2xy = 0$