(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

SPECIAL/SUPPLIMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **MAP 223/MAT 206**

COURSE TITLE: **ALGEBRAIC STRUCTURES II**

DATE: 20/01/2022 **TIME: 2 PM -4 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a. Define the following

i. Complementary relation (2 marks)

ii. Inverse relation (2 marks)

iii. Mathematical induction (2 marks)

b. Let $\mathbb{R} \subseteq X \times Y$be a binary relation from X to Y. Let $A, B \subseteq X$be subsets. Show that ;

i. If $A \subseteq B$then $R(A) \subseteq R(B)$ (4 Marks)

ii. $R(A \cup B) = R(A) \cup R(B)$ (6 Marks)

c. Prove that this rule of exponents is true for every natural number n:$(ab)^n = a^n b^n$

(7 marks)

d. Determine whether 225 is divisible by 2,3,4,5,6.9 and 10 (7marks).

QUESTION TWO (20 MARKS)

a. Define the following

i. Prime number (2 marks)

ii. Fundamental theorem of arithmetic (2marks)

iii. Greatest common divisor (2 marks)

b. Show that a composite integer n has a prime factor less than or equal to $\sqrt{n}$

(5 marks)

c. Show that there is an infinite number of prime numbers (6 marks)

d. Show that for positive integers a and b we have $ab = gcd(a.b).lcm(a.b)$

(3 marks)

QUESTION THREE (20 MARKS)

a. Define the following

i. A real sequence (2 marks)

ii. Limit of a sequence (3 marks)

iii. Null sequence (2 marks)

iv. Bounded sequence (2 marks)

b. Show that every convergent sequence is bounded (5 marks)

c. Show that the limit of a convergent sequence is unique (6 marks)

QUESTION FOUR (20 MARKS)

a. Define the following;

 i. Group (3 marks)

 ii. Abelian group (2 marks)

 iii. The subgroup criterion (2 marks)

b. Show that $(Z, +)$ is a group (4 marks)

c. Show that $[e, (1,2,3), (1,3,2)] \leq s_3$ (5 marks)

d. Let G be a group and $a \epsilon G$. Show that <a> is a subgroup of G. (4 Marks)

QUESTION FIVE (20 MARKS)

a. Define the following

 i. Ring (4marks)

 ii. Field (2marks)

b. State and prove the two properties of fields (8 marks)

c. Show that Z_3 is a field (3 marks)

d. State three properties of rings (3 marks)