(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR SCIENCE

COURSE CODE: **MAT 426**

COURSE TITLE: **FOURIER SERIES**

DATE: 20/01/2022 **TIME:** 2 PM - 10: AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30mks)

(a) (i) Find the complex Fourier Series $F_c(x)$ if $F_R(x) = \sin^3 2x$ (4mks)

(ii) Express the Real Fourier Series $F_R(x) = \cos^{-2} x$ in the Complex Fourier Series form $F_c(x)$ (8mks)

(b) If $D_n(\theta) = \frac{1}{2} + \cos\theta + \cos 2\theta + \ldots\ldots + \cos n\theta$, show that $\frac{2}{\pi}\int_0^{\pi} D_n(\theta)d\theta = 1$ (4mks)

(c) Compute the Fourier series of f defined by $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ 1 & 0 \leq x \leq \pi \end{cases}$ (6mks)

(d) Show that an even function can have no sine terms in its Fourier Series Expansion. 8mks)

QUESTION TWO (20mks)

(a) Find the Fourier Series expansion for $f(x) = \begin{cases} 0 & -\pi \leq x < 0 \\ \sin^2 x & 0 \leq x \leq \pi \end{cases}$ (14mks)

(b) If f belongs to R_2 on $\{-\pi \leq x \leq \pi\}$, show that for each n, $\Delta_n = \int_{-\pi}^{\pi} [F(x) - S_n(x)]^2\,dx$ is a minimum if for $S_n(x) = \frac{C_0}{2} + \sum_{k=1}^{n}(c_k \cos kx + d_k \sin kx)$ then $c_k = a_k and\ d_k = b_k$, where thea_k`$s$ $and$ $b_k$`s are the Fourier coefficients of f. (6mks)

QUESTION THREE (20mks)

(a) Using the Fourier Series for x^2, deduce $\frac{\pi^2}{6} = \sum \frac{1}{k^2}$ where $-\pi \leq x \leq \pi$ (15mks)

(b) If $s_n(x) = 2\sum_1^{\infty} \frac{(-1)^n \sin nx}{n}$ draw the graph of $S_2(x)$ (5mks)

QUESTION FOUR (20mks)

(a) Prove that (i) $\int_{-k}^{k} \sin\left(\frac{m\pi x}{k}\right) dx = \int_{-k}^{k} \cos\left(\frac{m\pi x}{k}\right) dx = 0$ (3mks)

(ii) $\int_{-k}^{k} \cos\left(\frac{m\pi x}{k}\right)\cos\left(\frac{n\pi x}{k}\right) dx = \int_{-k}^{k} \sin\left(\frac{m\pi x}{k}\right)\sin\left(\frac{n\pi x}{k}\right) dx = \begin{cases} O & m\neq n \\ k & m=n \end{cases}$

(14mks)

(b) If the series $f(x) = \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{l} + b_n \sin\frac{n\pi x}{l}\right)$ converges uniformly to f(x) in (-*l*.*l*), show that for n=1,2,3...........

(i) $a_n = \frac{1}{l}\int_{-l}^{l} F(x)\cos\frac{n\pi x}{l}dx$ (1mk)

(ii) $b_n = \frac{1}{l}\int_{-l}^{l} F(x)\sin\frac{n\pi x}{l}dx$ (1mk)

(iii) $A = \frac{a_o}{2}$ (1mk)

QUESTION FIVE (20mks)

(a) Find the Fourier Series expansion for $f(x) = \begin{cases} 0 & -\pi \leq x < 0 \\ \sin^2 x & 0 \leq x \leq \pi \end{cases}$ (14mks)

(b) If f belongs to R_2 on $\{-\pi \leq x \leq \pi\}$, show that for each n, $\Delta_n = \int_{-\pi}^{\pi} [F(x) - S_n(x)]^2\, dx$ is a minimum if for $S_n(x) = \frac{c_0}{2} + \sum_{k=1}^{n} (c_k \cos kx + d_k \sin kx)$ then $c_k = a_k and\ d_k = b_k$, where thea_k`$s\ and\ b_k$`s are the Fourier coefficients of f. (6mks)