(Knowledge for Development)

# KIBABII UNIVERSITY

## UNIVERSITY EXAMINATIONS

## 2020/2021 ACADEMIC YEAR

## SECOND YEAR SPECIAL/ SUPPLEMENTARY EXAMINATION

## FOR THE DEGREE OF BACHELOR OF SCIENCE

**COURSE CODE:** **STA 225**

**COURSE TITLE:** STATISTICS AND PROBABILITY

**DATE:** 18/01/2022 **TIME:** 8 AM -10 AM

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**INSTRUCTIONS TO CANDIDATES**

Answer Question One and Any other TWO Questions

TIME: 2 Hours

## QUESTION ONE [30MARKS]

a.) Let X be a random variable with probability distribution $f(x) = \begin{cases} \frac{2}{15} - \frac{2x}{225}, & 0 < x < 15 \\ o, & elsewhere \end{cases}$

i.) Show is that f (x) is a continuous P.d.f. [3marks]

ii.) Find the expected value of X [3marks]

iii.) Find the variance of X [4marks]

b.) Let the probability density function of X be given by

$$f(x) = \begin{cases} k\left(\frac{1}{3}\right)^x, & x = 0,1,2,3, \ldots. \\ o, & otherwise \end{cases}$$

i.) Find the value of k [4marks]

ii.) Find the moment generating function of X. [6marks]

iii.) Using the m.g.f. find the mean of X. [4marks]

c.) Let X be bin (2,p) and let Y be bin(4,p).If $\Pr(X \geq 1) = \frac{5}{9}$,find $\Pr(Y \geq 1)$.[6marks]

## QUESTION TWO [20MARKS]

a.) Suppose that the PH of soil samples taken from a certain region is normally distributed with mean PH of 6.0 and standard PH of 0.1.If the PH of a randomly selected soil sample from this region is determined, what is the probability that :

i.) The resulting PH is between 5.90 and 6.15? [4marks]

ii.) The resulting PH is at most 5.95? [4marks]

b.) The random variable X has a probability density function given by

$$f(x) = \begin{cases} \frac{1}{\sqrt{2\pi}} e^{\frac{-(x-1)^2}{2}}, & -\infty < x < \infty \\ o, & elsewhere. \end{cases}$$

Show that the moment generating function of X is given by $M_x(t) = e^{t+\frac{t^2}{2}}$

[6marks]

c.) Let $\psi(t) = \ln[m(t)]$, where m(t) is the moment generating function of a distribution. Prove that $\psi^{I}(0) = \mu$ and $\psi^{II} = \sigma^2$. [6marks]

## QUESTION THREE [20MARKS]

Let X and Y be independent and identically distributed Poisson random variables, that is f(x) = $\begin{cases} \frac{e^{-\lambda}\lambda^x}{x!}, x = 0,1,2, .... \\ o, \ elsewhere \end{cases}$ and f(y) = $\begin{cases} \frac{e^{-\lambda}\lambda^y}{y!}, y = 0,1,2, .... \\ o, \ elsewhere \end{cases}$

i.) Given that Z = X+Y, find the m.g.f. of Z. [12marks]

ii.) Hence calculate E(Z) [3marks]

iii.) Hence find Var(Z) [5marks]

## QUESTION FOUR [20MARKS]

a.) Define the probability density function,f(x) for a binomial random variable X,with parameters n and p. [2marks]

b.) Let X be a random variable having binomial distribution with parameters n=100 and p=0.1.Evaluate Pr[X $\leq \mu - 3\sigma$] [4marks]

c.) The probability distribution of a discrete random variable X is given by

f(x) = f(x) = $\begin{cases} k\binom{3}{x}\binom{4}{3-x}, \ x = 0,1,2,3. \\ o, \ otherwise \end{cases}$ where k is a constant.

i.) Show that k = $\frac{1}{35}$. [4marks]

ii.) Hence find the mean and variance of X. [10marks]

## QUESTION FIVE [20MARKS]

a.) A random variable X has a probability density function given by

f(x) = $\begin{cases} (qx^2), & 0 < x < 1 \\ o, & otherwise \end{cases}$. Given that E(X) = $\frac{2}{3}$, determine the value of q.

[8marks]

b.) 250 watches are inspected and the numbers of defective per set are recorded.

| Number of defective | Number of sets |
|---|---|
| 0 | 10 |
| 1 | 90 |
| 2 | 82 |
| 3 | 35 |
| 4 | 43 |
| 5 | 20 |

i.) Estimate the average number of defective per set and expected frequency of 0, 1,2,3,4 and5. [4marks]

ii.) Fit a Poisson distribution of the data. [8marks]