(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: MAT 222/MAA 221

COURSE TITLE: CALCULUS III

DATE: 18/01/2022 **TIME:** 8 AM -10 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) Find the domain and range for the function $f(x, y) = \sqrt{2y - x}$ (2 mks)

b) Find the radius and interval of convergence of the series

$\sum_{k=1}^{\infty} \frac{(-4)^k (x-6)^k}{k 4^{k-1}}$ (5 mks)

c) The production function is given by $f(x, y) = 4xy$ maximize the this function subject to budget constraint $5x + 4y = 7$ (5 mks)

d) Find the volume in the 1st octant between the planes $z = 0$, and $z = 3x + y - 2$ And inside the cylinder $x^2 + y^2 = 25$ (5 mks)

e) Investigate the convergence of $\sum_{k=0}^{\infty} \frac{(-2)^k e^k}{k!}$ (6 mks)

f) Locate any relative extreme points and determine their nature for the function $f(x, y, z) = 5x^2 + 3y^2 + z^2 - 12x + 18y - 5z + 40$ (7 mks)

QUESTION TWO (20 MARKS)

a) Use the Lagrange multipliers to find the local extrema of the function $f(x, y) = y^3 - 9x^2$ Subject to $x^2 + y^2 = 16$ (7 mks)

b) Suppose x is a differentiable function near each (y, z) for the equation $z\sin y - 4x^2 z + 4e^{xyz} = 33$ find $\frac{\partial x}{\partial y}$ (3mks)

c) Evaluate $\lim_{(x,y)\to(4,4)} \frac{\sqrt{x}-\sqrt{y}}{y^2-xy}$ (4 mks)

d) A closed cylinder has a surface area of $82.62\ cm^2$ Find the radius and the height that will yield minimum volume (6 mks)

QUESTION THREE (20 MARKS)

a) Let $z = e^{2x} cosec y$ and $x = 4s^2t^2 - t$ and $y = s^3 - 3t$ find $\frac{\partial z}{\partial t}$ and $\frac{\partial z}{\partial s}$ (7 mks)

b) If $R = \left\{x, y \,/\, 0 \le x \le 1 \text{ and } 1 \le y \le 2\right\}$ evaluate $\iint_R (-16xy^2 - x^3y + 8)dA$ (3 mks)

c) Find the volume of the solid bounded by the graphs of $z = 9 - y^2$, $x - z = 2$, $x = 0$, and $z = 0$ (5 mks)

d) Consider the series $S_n = \frac{1}{\sqrt{2n-1}}$ using the integral test, determine whether the series converges or diverges (5 mks)

QUESTION FOUR (20 MARKS)

a) Consider the series $\sum_{k=1}^{\infty} \frac{(-3)^{k-1}4^k}{3k^2}$ use ratio theorem to show that the series diverges (4 mks)

b) Locate and classify all critical points of

$f(x_1, x_2, x_3) = 2x_2 + 6x_1x_3 - 2x_1^3 - x_2^2 - 6x_3^2 - 16$ (6 mks)

c) For what values does the series converge $\sum_{n=1}^{\infty} \frac{(x-5)^n}{n}$ (4 mks)

d) Verify that the Tailor series expansion for the function $f(x) = sinx$ about $x = 0$ is $sinx = \sum_{n=0}^{\infty} \frac{(-1)^n (x)^{2n+1}}{(2n+1)!}$ hence find the Maclaurin series for $f(x) = xsinx$ (6 mks)

QUESTION FIVE (20 MARKS)

a) Use the 1st principles to determine $\frac{\partial f}{\partial y}$ given that $f(x, y) = xy - 2x^2y^2 - 2y^3$ (4 mks)

b) Let $f(x, y, z) = 2xln(xz) - e^{x^2y} + 3\cos(xyz)$. Find

(i) f_{xx} (2 mks)

(ii) f_{yyz} (3 mks)

c) Evaluate $\int_0^{\pi} \int_0^{\frac{\pi}{2}} \int_0^{siny} 4sinzcosydxdydz$ (5 mks)

d) Find the area of the portion of the cone $x^2 + y^2 = 9z^2$ lying above the xy-plane and inside the cylinder $x^2 + y^2 = 5y$ (6 mks)