(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **STA 221/STA 242**

COURSE TITLE: PROBABILITY AND DISTRIBUTION MODELS

DATE: 19/01/2021 **TIME:** 2:00 PM - 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE COMPULSORY (30MARKS)

a) Let X and Y be two discrete random variables with a joint probability function f(x, y). Define

   i. Marginal probability function of X

   ii. Marginal probability function of Y (2marks)

b) The distance between flaws on a long cable is exponentially distributed with mean 2m

   i. Find the probability that the distance between two flows is greater than 15m (3marks)

   ii. Find the probability that the distance between two flaws is between 8 and 20 (4marks)

c) Given the joint density of two continuous random variables X and Y as shown below. Find the marginal and conditional p.d.f of X and Y (8marks)

$$f(x,y) = \begin{cases} \frac{1}{4}(1+xy)|x| < 1, |y| < 1 \\ 0 \qquad \text{Otherwise} \end{cases}$$

d) Consider an experiment of tossing the tetra-hedra (regular four sided polyhedron) each with sides labelled 1-4. Let X be the score on the 1st tetrahedron and X_1 be the score on the 2nd tetrahedron. Let Y denote the maximum of X and X_1 . Find the joint C.D.F of

   i. F(1, 2) (2marks)

   ii. F(4,3) (2marks)

e) Let X and Y have a bivariate density

$$f(x,y) = \begin{cases} e^{(-x-y)} & x > 0, y > 0 \\ 0 & \text{Otherwise} \end{cases}$$

Show that $f(x,y)$ is a p.d.f (3marks)

f) Let X and Y be jointly distributed random variables with joint p.d.f as shown below. Obtain

$$f(x,y) = \begin{cases} p^{(x+y)}(1-p)^{2-x-y} & x = 0,1, y = 0,1 \\ 0 & \text{otherwise} \end{cases}$$

   i. E(X)

   ii. E(X+Y)

   iii. E(X-Y) (6marks)

QUESTION TWO (20MARKS)

The joint probability function of two discrete random variables X and Y is given by $f(x,y) = c(2x+y)$, where x and y assume all integers such that $0 \le x \le 2$, $0 \le y \le 3$, and $f(x,y) = 0$ otherwise. The sample points (x,y) for which probabilities are different from zero are indicated in the table below. The probabilities associated with these points, given by $c(2x+y)$, are shown in **Table below.**

X \ Y	0	1	2	3	Totals ↓
0	0	c	$2c$	$3c$	$6c$
1	$2c$	$3c$	$4c$	$5c$	$14c$
2	$4c$	$5c$	$6c$	$7c$	$22c$
Totals →	$6c$	$9c$	$12c$	$15c$	$42c$

a) Find the value of the constant c. (2marks)

b) Find $P(X=2, Y=1)$. (1marks)

c) Find $P(X \ge 1, Y \le 2)$. (1marks)

d) Find the marginal probability function of X and Y. (4marks)

e) Show that the random variables X and Y are independent. (3marks)

f) Evaluate $E(X)$, $E(Y)$, $E(X,Y)$, $E(X^2)$, $E(Y^2)$, $Var(X)$, $Var(Y)$ and $Cov(X,Y)$ (9marks)

QUESTION THREE (20MARKS)

a) Given the joint density of X_1 and X_2 as

$$f(x,y) = \begin{cases} 1 & 0 < x_1 < 1 \quad 0 < x_2 < 1 \\ 0 & Otherwise \end{cases}$$

Find

i. The joint density of $Y = X_1 + X_2$ and $Z = X_2$ (4marks)

ii. The marginal density of Y (3marks)

b) Show that the variance of the Exponential distribution is $1/\lambda^2$ (13marks)

QUESTION FOUR (20MARKS)

a) The joint probability density function of the thickness X and hole diameter Y(bolt in mm) of a randomly chosen washer is

$$f(x,y) = \begin{cases} \frac{1}{6}(x+y) & 1 \le x \le 2, 4 \le y \le 5 \\ 0 & Otherwise \end{cases}$$

i. Find the conditional probability density of Y given X= 1.2 (7marks)

ii. Find the probability that the hole diameter is less than or equal to 4.8 given that the thickness is 1.2mm (2marks)

iii. Find the conditional expectation of y given that X = 1.2 (2marks)

iv. Find the value of E [Y]?. Does it differ from E[Y/X= 1.2] (3marks)

b) Suppose that X and Y are the two discrete random variables with the joint p.d.f given by

$$f(x,y) = \frac{1}{54}(x+y) \quad x = 1,2,3 \text{ and } y = 1,2,3,4$$

Determine the conditional distribution for Y given that X=x, hence calculate

i. P(y=1/x=1)

ii. P(Y=4/X=3) (6marks)

QUESTION FIVE (20MARKS)

a) Suppose that *Y*1 and *Y*2 are random variables (discrete or continuous). Show that
V (Y1 + Y2) = V (Y1) + V (Y2) + 2Cov (Y1, Y2) (4marks)

b) A small health-food store stocks two deferent brands of grain. Let X denote the amount of brand 1 in stock and let Y denote the amount of brand 2 in stock (both X and Yare measured in 100s of lbs). The joint distribution of X and Y is

$$f_{x,y}(x,y) = \begin{cases} 24xy & x > 0, y > 0, 0 < x + y < 1 \\ 0 & Otherwise \end{cases}$$

i. Find the conditional pdf $f_{x/y}(x/y)$. (5marks)

ii. Compute P(X>0.5/Y = 0.3). (4marks)

iii. What is the variance for the total amount of grain in stock? (7marks)

END