(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: STA 444

COURSE TITLE: SEQUENTIAL ANALYSIS

DATE: 19/01/2022 **TIME:** 11:00 AM -1:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

Question one (30mks)

(a) Distinguish between sequential analysis and sequential tests (4mks)

(b) State the likelihood ratio for testing H_0: $\theta = \theta_0$ against H_1: $\theta = \theta_1$ where $\theta_0 \leq \theta_1$ (4mks)

(c) Given that $\alpha = 0.05$ and $\beta = 0.01$, find the conditions under which the alternative hypothesis H_1 will be accepted (6 mks)

(d) Given $x_1, x_n, \ldots\ldots\ldots\ldots\ldots x_n$ is a random sample from $f(x, \theta)$, $\theta \in \Omega$ with illustrations show how

i. The estimate of the parameter can be derived (4mks)

ii. The sequential probability ratio test can be carried out (4mks)

(e) Examine if a test exists for testing H_0: $\mu = \mu_0$ in $f(x, \mu) = e^{-(x-\mu)}$ if $\mu < x < \infty$ (8 mks)

Question Two (20 marks)

Given the exponential distribution

$f_2(x) = \theta^{-1} \exp(-x/\theta)$ where $\theta_0 \leq \theta_1$

State that H_0 will be accepted immediately

$\bar{x} \leq S_1$ where $S_1 = n\left(\theta_0 + n\log\frac{\theta_1}{\theta_0}\right)\frac{\theta_0\theta_1}{\theta 1 - \theta 0}$

Question Three (20 marks)

Derive the value of K in SPRT given

$X \sim N(\mu, \delta^2)$ where δ^2 is know with a one sided alternative hypothesis

Question four (20 marks)

(a) How can the expected value of the stopping time in sequential analysis be obtained? (4 mks)

(b) Distinguish between α and β as used in hypotheses testing (2 mks)

(c) With an illustration briefly explain the critical region (4 mks)

(d) State the characteristics that are necessary in sequential analysis in order accept or reject H_0 (4 mks)

(e) Highlight the significance of ASN functions (2 mks)

(f) Show how the expected sample size in a sequential test can be derived (4 mks)

Question five (20 marks)

(a) What is the power function of parameter θ (2 mks)

(b) Let $x_1, x_2, \ldots\ldots\ldots x_n$ be iid random variables with a common distribution p. state the stage when SPRT stops sampling in testing $H_0: p = p_0$ vs $H_1: p = p_1$ (4 mks)

(c) Given X $\sim \beta(n, p)$.obtain the test for: $\mathbf{p \leq p_0}$ **vs** H_1: $p \geq \mathbf{p_0}$ (8 mks)

(d) Given that $\Phi_0 \leq \bar{x} \leq \Phi_1$, State the Walds approximation for $\Phi_0 and\ \Phi_1$ and find $E(T|H_0)$and $E(T|H_1)$ (6 mks)