(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: **MAP 311**

COURSE TITLE: **REAL ANALYSIS II**

DATE: 17/05/2022 **TIME:** 2:00 PM - 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) Define the following terms:

(i) Open and closed set (4 marks)

(ii) An interior point (2 marks)

(iii) Cauchy sequence (2 marks)

(iv) Compact set (4 marks)

b) State Uniform Continuity Theorem (2 marks)

c) Show that for every $x \in X$ and r > 0 the open ball B(x, r) in a metric space is open. (10 marks)

d) Suppose f : X→ Y and g : Y→ Z are continous between metric spaces, then the composition g ₒ f : X→ Z is continous. (6 marks

QUESTION TWO (20 MARKS)

a) Define the following terms:

(i) Metric space (5 marks)

(ii) Open and closed ball (5 marks)

b) Show that if (X_i , d_i) where I = 1, 2,…,n are metric spaces, then X = $X_1 \times X_2 \times \ldots \times X_n$ become a metric space with the metric d defined by:

d (x , y): = $\sum_{i=1}^{n} d_i (x_i, y_i)$ for all x = ($x_i, \ldots, x_n$) and y = ($y_i, \ldots, y_n$) (10 marks)

QUESTION THREE (20 MARKS)

a) Define the following terms:

(i) Accumulation point (4 marks)

(ii) Relatively compact sets (2 marks)

(iii) Open cover (2 marks)

(iv) Total boundedness (2 marks)

b) Let (X, d) be a metric space. Show that closed subsets of compact metric spaces are compact. (10 marks)

QUESTION FOUR (20 MARKS)

a) Define the following terms:

(i) Continous function (5 marks)

(ii) Uniform continuity (5 marks)

b) Show that for every non-empty set $A \subseteq X$ the map $X \rightarrow \mathbb{R}$, $x \mapsto \text{dist}(x, A)$ is continous. (10 marks)

QUESTION FIVE (20 MARKS)

a) Show that a sequence in a metric space (X, d) has at most one limit. (10 marks)

b) Show that if (x_n) is a sequence in a metric space (X, d) and $x_0 \in X$, then the following statements are equivalent:

(1) $\lim_{n\to\infty} x_n = 0$

(2) For every $\varepsilon > 0$, $\exists$ $n_0 \in \mathbb{N}$ such that d (x_n, x_0) $< \varepsilon$ for all n $\geq n_0$. (10 marks)