(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE MATHEMATICS

COURSE CODE: **MAP 121**

COURSE TITLE: **ALGEBRAIC STRUCTURES I**

DATE: 17/05/2022 **TIME:** 2:00 PM - 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE COMPULSORY (30 MARKS)

a) Define the following

i. Group (4marks)

ii. Normal Subgroup (2marks)

iii. Binary operation (2marks)

iv. Field (2marks)

b) Define the Klein four group S_3 and give its subgroups . (8marks)

c) If G is the quarternion group and its subgroup $H = \{\pm 1\}$, give the left cosets of H in G (5marks)

d) Generate a 3×3 circulant matrix starting with $[a, b, c]$ (3marks)

e) Show that every subgroup of an abelian group is normal (4marks)

QUESTION TWO (20 MARKS)

a) Define the following

i. Cyclic Group (2marks)

ii. Quotient Group (2marks)

iii. Symmetric group (2marks)

b) Draw the cayley table for the quaternion group (8marks)

c) Find the inverse of the following matrix, whose entries are elements of $\mathbb{Z}_6$ (6marks)

$$A = \begin{bmatrix} 5 & 3 \\ 4 & 2 \end{bmatrix}$$

QUESTION THREE (20 MARKS)

c) Define the following

i. Ring (5marks)

ii. Bijective functions (2marks)

d) Let $H \leq G$ and $x, y \in G$ then proof that either $xH = yH$ or $xH \cap yH = \emptyset$ (6marks)

e) Let $*$ be an associative binary operation on a set S. Then for all $a \in S$ and all natural numbers m and n, show that

i. $a^m * a^n = a^{m+n}$ (2marks)

ii. $(a^m)^n = a^{mn}$ (2marks)

f) State 3 examples of fields (3marks)

QUESTION FOUR (20 MARKS)

a) Define the following

   i. Simple group (2marks)

   ii. Index of a group (2marks)

   iii. Proper subgroup (2marks)

b) Define the klein-4 Group (6marks)

c) Prove that the quotient group $G/_H$ satisfies the group axioms (5marks)

d) Give examples of simple groups (3marks)

QUESTION FIVE (20 MARKS)

a) State and proof the lagranges theorem (6marks)

b) State five examples of binary operations (5marks)

c) Show that if $|G| = p$ where p is a prime, then G is cyclic (4marks)

d) State the subgroups of the Quarternion group (5marks)