(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

SUPPLEMENTARY/SPECIAL EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE MATHEMATICS

COURSE CODE: **MAP 321/MAT 302**

COURSE TITLE: **REAL ANALYSIS III**

DATE: 21/01/2022 **TIME:** 11:00 AM - 1:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE COMPULSORY (30 MARKS)

a) Define the following

i. Bounded set (2marks)

ii. Integrable function (2marks)

iii. Lebesque integral (2marks)

iv. Infinite series (2marks)

v. Natural logarithms (2marks)

b) Show that suppose $f_n$and f are functions defined on an interval J. If there exists a sequence (x_n) in J such that $|f_n(x_n) - f(x_n)| \nrightarrow 0$, then (f_n) does not converge uniformly to f on J. (5marks)

c) Show that the logarithm of a product of two positive numbers is the sum of their logarithm (5marks)

d) State the monotone convergence theorem (4marks)

e) Suppose (f_n) is a sequence of continuous functions defined on an interval [a,b] which converges uniformly to a function f on [a,b] then show that f is continuous and $\lim_{n\to\infty} \int_a^b f_n(x)dx = \int_a^b f(x)dx$ (6marks)

QUESTION TWO (20 MARKS)

a) Show that the series $1 + r + r^2 + \dots + r^N = \frac{r^{N-1} -1}{r-1}$ (4marks)

b) Show that if f is increasing on $[a, b]$, then f is of bounded variation on $[a, b]$ and $V(f, [a, b]) = f(b) - f(a)$ (4marks)

c) Show that if $f: [a, b] \to \mathbb{R}$ is a function, let $\{x_i | 0 \leq i \leq n\}$ be a partition of $[\mathrm{a}, \mathrm{b}]$ and let $\{y_i | 0 \leq i \leq m\}$ be a partition of $[\mathrm{a}, \mathrm{b}]$ such that $\{x_i | 0 \leq i \leq n\} \subseteq \{y_i | 0 \leq i \leq m\}$ then $\sum_{i=1}^{n} |f(x_i) - f(x_{i-1})| \leq \sum_{i=1}^{m} |f(y_i) - f(y_{i-1})|$ (8marks)

d) Show that if the series $\sum_{n\geq 1} a_n$ converges then as $a_n \to 0$ as $n \to \propto$ (4marks)

QUESTION THREE (20 MARKS)

a) Define the following

i. Dominated series (3marks)

ii. A partition (2marks)

iii. Uniform convergence (2marks)

iv. Bounded variation (3marks)

b) Show that the infinite series $\sum_{n\geq 0} x^n$ converges if $|x| < 1$ and diverges if $|x| > 1$ (4marks)

c) Show that assuming F is an increasing step function on I so that $F(t) = \sum_{i=1}^{N} a_i I\{t \leq t_i\}$ with $t_0 = \min(I) < t_1 < t_2 ... < t_N = \max(I)$ and $a_i \geq 0$ and if g is continuous, then $\int g(x)dF(x) = \sum_i^N g(t_i)a_i$ (6marks)

QUESTION FOUR (20 MARKS)

a) Define;

i. a Fourier series (2marks)

ii. Finite series (2marks)

iii. Convergent series (2marks)

iv. Riemann-Stieltjes integral (3marks)

v. Exponential function (2marks)

b) State the Fatou's lemma (4marks)

c) If f is a bounded function defined on [a; b] such that f is Riemann integrable, then f is Lebesgue integrable and $(R)\int_a^b f(x)\,dx = \int_{[a,b]} f(x)dx$ (5marks)

QUESTION FIVE (20 MARKS)

a) Assume that the sequence $\{f_n\}$ converges uniformly to g, show that $\{f_n\}$ converges pointwise and that $f = g$ (5marks)

b) Show that the Fourier coefficient $f_n \to 0$ as $|n| \to \infty$ (4marks)

c) Assume that the sequence $\{f_n\}$ is a sequence of continuous functions which converges uniformly to f on I, show that f is continuous. (5marks)

d) Suppose f_n is a sequence of continuous functions defined on an interval J which converges uniformly to a function f, show that f is continuous on J(6marks)