

(Knowledge for Development)

# **KIBABII UNIVERSITY**

UNIVERSITY EXAMINATIONS

**2020/2021 ACADEMIC YEAR** 

SECOND YEAR SECOND SEMESTER

SPECIAL/SUPPLIMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND **BACHELOR OF SCIENCE** 

COURSE CODE:

**MAT 224** 

COURSE TITLE: ANALYTIC GEOMETRY

DATE:

20/01/2022

TIME: 11:00 AM - 1:00 PM

# INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

### **QUESTION ONE (30 MARKS)**

- a) Convert the parametric equation  $x = 3 + \sqrt{3}\cos\theta$  and  $y = -2 + \sqrt{3}\sin\theta$ , into Cartesian form, and hence identify the curve (3 marks)
- b) If  $y = \left(-\frac{4}{9} + x\right)^{\frac{3}{2}}$ . Find the length of the curve between y = 0, and y = 1 (3 marks)
- c) A point P divides a line through (3, 1, 4) and (8, 11, 9) in the ratio 3:2. Find coordinates of the point Q. (3 marks)
- d) Show that the planes 2x 4y + 3z + 6 = 0 and -6x + 12y 9z 2 = 0 are parallel and hence find the shortest distance between them (Give your answer to two decimal places.

  (4 marks)
- e) Find the angle between the lines whose direction ratios are (2, 3,1) and (3, 4, 2). (3 marks)
- f) Sketch the curve whose equations are  $(x + 4)^2 = -12(y + 1)$ . (4 marks)
- g) Identify the surface for each of the following equations. (4 marks)
  - i.  $r^2 + z^2 = 100$
  - ii. z=r
- h) Perform each of the following conversion
  - i. Cylindrical coordinates  $(\sqrt{6}, \frac{\pi}{4}, \sqrt{2})$  to spherical coordinates (3 marks)
  - ii. Cartesian coordinates  $(-1,1,-\sqrt{2})$  to spherical coordinates (3 marks)

### **QUESTION TWO (20 MARKS)**

A plane P has three points (2, 5, 6), (2, 1, 8) and (3, 4, 7). The plane intersects with a line through points (2, 1, 4) and (4, 3, 8) at point M. Find;

a) the equation of the plane

(5 marks)

b) equation of the line in parametric form

(3 marks) (3 marks)

c) the coordinates of Md) the ratio in which the plane divides the line

(5 marks)

e) any vector parallel to the plane

## (4 marks)

(3 marks)

### **QUESTION THREE (20 MARKS)**

- a) Given the equation of an ellipse  $9x^2 18x + 25y^2 + 100y = 116$ 
  - i. write the equation in standard form
    - find the foci, center, the vertices of the curve and hence sketch it (5 marks)
  - iii. if the ellipse was translated 5 units to the right and 2 units up, what would be its new equation? (2 marks)
- b) Two ellipses  $4x^2 + 5y^2 = 81$  and  $5x^2 + 4y^2 = 81$  intersect at different points. Find the points of intersection (5 marks)
- c) Sketch the hyperbola  $\frac{x^2}{16} \frac{y^2}{9} = 1$  and its asymptotes (5 marks)

#### **QUESTION FOUR (20 MARKS)**

- a) Using a simple sketch, derive the relationship between
  - i. Cartesian and spherical coordinates
  - ii. Cartesian and cylindrical coordinates
- b) Carry out the following conversion, simplify your answers
  - The Cartesian equation  $x^3 + 2x^2 6z = 4 2y^2$  to cylindrical coordinates
    (3 marks)

(4 mark)

(4 marks)

- ii. The Cartesian equation  $x^2 + y^2 = -z^2 2$ to cylindrical coordinates (3 marks)
- iii. The Cartesian coordinates (2, 1, 4) to cylindrical and spherical coordinates (6 marks)

#### **QUESTION FIVE**

- (a) Given lines whose direction ratios are given by the relations l + m + n = 0 and  $l^2 + m^2 n^2 = 0$ , find the angle between the lines. (8marks)
- (b) Find the equation of the locus of the point whose distance from the point (2, -2, 2) is two times its distance from the plane 2x + 3y 6z = 12 (5marks)
- (c) Find the coordinates of the point where a line through (3, -4, -5) and (2, -3, 1) crosses the plane 2x + y + z = 7 (7marks)