(Knowledge for Development)

KIBABII UNIVERSITY

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UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

(CHEMISTRY AND PHYSICS)

COURSE CODE: **STA 142**

COURSE TITLE: INTRODUCTION TO PROBABILITY

DATE: 17/05/2022 **TIME**: 9:00 AM - 11:00 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

1. (a) Define the following terms: Sample spaces, Events, Mutually exclusive events (3 mks)

   (b) Let A and B be two events. Suppose that $P(A) = 0.4$, $P(B) = 0.5$, and $P(A \cap B) = 0.1$. Find the probability that A or B occurs, but not both. (3 mks)

   (c) In the table below, probabilities for the number of times a computer system crashes in a week is presented. Let A be the event that there are more than two crashes during the week, and let B be the event that the system crashes at least once. Find the sample space and the subsets that correspond to A and B. Then find $p(A)$ and $p(B)$ (3 mks)

No.of crashes	0	1	2	3	4
Probability	0.60	0.30	0.05	0.04	0.01

   (d) A shop receives a batch of 1000 cheap lamps. The odds that a lamp is defective are 0.1%. Let X be the number of defective lamps in the batch.

      i. What kind of distribution does X have? What is/are the value(s) of parameter(s) of this distribution? (2 mks)

      ii. What is the probability that the batch contains no defective lamps? One defective lamp? More than two defective ones? (3 mks)

   (e) Let X be a random variable with the probability distribution given as:

x	0	1	2	3	4	5	6	7	8
p(x)	a	2a	3a	4a	5a	6a	7a	8a	9a

      i. Determine the value of a (2 mks)

      ii. Find $P(0 < X < 5)$ (3 mks)

   (f) The letters of the word RANDOMLY are written, one on each of the eight separate cards. The cards are laid out in a line. Find the probability that the vowels are all placed together. (4 mks)

   (g) In a laboratory, two experiments are repeated every day of the week in different rooms until at least one is successful, the probability of success being p for each experiment. Supposing that the experiments in different rooms and on different days are performed independently of each other, what is the probability that the laboratory scores its first successful experiment on day n? (3 mks)

   (h) A fair die is thrown twice. A is the event "sum of the throws equals 4," B is "at least one of the throws is a 3." Calculate $P(A|B)$; Are A and B independent? (4 mks)

QUESTION TWO (20 MARKS)

2. (a) Let X be random variable with pdf

$$f(x) = \begin{cases} \frac{x}{6}, & x = 0, 1, 2, 3 \\ 0, & elsewhere \end{cases}$$

Compute;

i. $E(X)$, (2 mks)

ii. $E(3X)$ (2 mks)

iii. $Var(X)$ (4 mks)

(b) We toss a coin three times. For this experiment we choose the sample space $\Omega = \{HHH, THH, HTH, HHT, TTH, THT, HTT, TTT\}$ where T stands for tails and H for heads.

i. Write down the set of outcomes corresponding to each of the following events:
A : "we throw tails exactly two times." (2 mks)
B : "we throw tails at least two times." (2 mks)
C : "tails did not appear before a head appeared." (2 mks)
D : "the first throw results in tails." (2 mks)

ii. Write down the set of outcomes corresponding to each of the following events: A^c, $A \cup (C \cap D)$, and $A \cap D^c$. (5 mks)

QUESTION THREE (20 MARKS)

3. (a) A university library has five copies of a textbook to be used in a certain class. Of these copies, numbers 1 through 3 are of the 1st edition, and numbers 4 and 5 are of the 2nd edition. Two of these copies are chosen at random to be placed on a 2-hour reserve.

i. Write out an appropriate sample space S. (2 mks)

ii. Consider the events A, B, C, and D, defined as follows, and express them in terms of sample points.
A = both books are of the 1st edition, (2 mks)
B = both books are of the 2nd edition, (2 mks)
C = one book of each edition, (2 mks)
D = no book is of the 2nd edition. (2 mks)

(b) Let X have the pdf

$$f(x) = \begin{cases} \frac{1}{2}(x+1), & -1 < x < 1 \\ 0, & elsewhere \end{cases}$$

Find

i. $Var(X)$ (5 mks)

ii. $Var(5X)$ (5 mks)

QUESTION FOUR (20 MARKS)

4. (a) Consider tossing two fair dice. Let X denote the sum of the upturned values of the two dice and Y their absolute difference. Calculate the expected value of X and Y. (8 mks)

   (b) Let X (in tonnes) be a random variable representing the quantity of sugar sold in a day at a certain factory with a distribution function as shown;

$$f(x) = \begin{cases} cx, & 0 \leq x \leq 3 \\ c(10-x), & 3 < x \leq 5 \\ 0, & elsewhere \end{cases}$$

   i. Find c such that $f(x)$ is a pdf (4 mks)

   ii. Find $P(X \leq 3)$ (2 mks)

   iii. Find $P(X > 3)$ (2 mks)

   iv. Find $P(2.5 \leq X \leq 5)$ (4 mks)

QUESTION FIVE (20 MARKS)

5. (a) A breath analyzer, used by the police to test whether drivers exceed the legal limit set for the blood alcohol percentage while driving, is known to satisfy $P(A|B = P(A^c|B^c) = p$, where A is the event breath analyzer indicates that legal limit is exceeded and B drivers blood alcohol percentage exceeds legal limit. On Saturday night about 5% of the drivers are known to exceed the limit.

   i. Describe in words the meaning of $P(B^c|A)$. (2 mks)

   ii. Determine $P(B^c|A)$ if $p = 0.95$. (4 mks)

   iii. How big should p be so that $P(B|A) = 0.9$? (4 mks)

   (b) The faculty in an academic department in Kibabii University consists of 4 assistant professors, 6 associate professors, and 5 full professors. Also, it has 30 graduate students. An ad hoc committee of 5 is to be formed to study a certain curricular matter.

   i. What is the number of all possible committees consisting of faculty alone? (3 mks)

   ii. How many committees can be formed if 2 graduate students are to be included and all academic ranks are to be represented? (4 mks)

   iii. If the committee is to be formed at random, what is the probability that the faculty will not be represented? (3 mks)