(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

SPECIAL/SUPPLIMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: MAA 222/MAT 322

COURSE TITLE: OPERATIONS RESEARCH I

DATE: 20/01/2022 **TIME: 8:00 AM – 10:00 AM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MKS)

a) What is linear programming? What types of problems can linear programming help in solving? What characteristics must a problem have if linear programming is to be used? (8 marks)

b) . Why is the simplex method considered superior to the graphic method? Explain. (2 marks)

c) What are the comparisons between transportation problem and assignment problems (4 marks)

d) Form the dual of the following LP problem.

Maximize $z = 5x_1 + 6x_2$

Subject to

$$4x_1 + 7x_2 \leq 20$$
$$5x_1 + 2x_2 \leq 10$$
$$6x_1 + 8x_2 \leq 25$$

x_1 *and* x_2 are unrestricted in sign. (3 marks)

e) A farmer requires to feed his pigs as cheaply as possible. The pigs requires the diet consisting of a minimum amount of three nutrients N_1,N_2 and N_3 which form a part of the commercially available food stuffs F_1, F_2, F_3 and F_4.The number of units contained in each food stuff, cost per unit of each and minimum requirements are given below

Nutrients	Food stuffs				Minimum Requirements
	F_1	F_2	F_3	F_4	
N_1	5	8	4	1	50
N_1	3	8	7	5	40
N_3	4	0	5	4	8
Cost/unit	1.0	0.9	1.2	0.9	

(i) set up a linear programming model for this problem (3 marks)

(ii) hence solve it using the dual simplex method (10 marks)

QUESTION TWO (20 MKS)

Use the branch and bound technique to obtain an optimum assignment schedule from the following cost matrix concerning the problem of assigning 5jobs to 5persons.

	PERSONS				
JOBS	1	2	3	4	5
A	8	4	2	6	1
B	0	9	5	5	4
C	3	8	9	2	6
D	4	3	1	0	3
E	9	5	8	9	5

QUESTION THREE (20 MKS)

a) Define Model, discuss the steps of Modelling

Solve the following LP problem using Simplex Method.

Maximize $z = 6x_1 + 8x_2$

Subject to

$$5x_1 + 10x_2 \leq 60$$

$$4x_1 + 4x_2 \leq 40$$

$$x_1 \quad and \quad x_2 \geq 0$$

(5 Marks)

b) Hence find the new solutions if

(i) the Right Hand side constants of the constraint 1 and constraint 2 are changed from 60 and 40 to 40 and 20 respectively.

(ii) the Right Hand side constants of the constraint 1 and constraint 2 are changed from 60 and 40 to 20 and 40 respectively.

(iii) Determine the range of optimality

If a new constraint $6x_1 + 3x_2 \leq 48$ is added. (15marks)

QUESTION FOUR (20 MKS)

a) Solve the following Linear programming problem using the Big M method.
Minimize $z = 24x_1 + 30x_2$

Subject to: $2x_1 + 3x_2 \geq 10$

$4x_1 + 9x_2 \geq 15$

$6x_1 + 6x_2 \geq 20$

$x_1 \quad and \quad x_2 \geq 0$

(8Marks)

b) In a multi-speciality hospital, nurses report to duty at the end of every four hours as shown in a table below. Each nurse, after reporting, will work for 8 hours continuously. The minimum number of nurses required during various periods are summarized in the table below. Develop a Mathematical Model to determine the number of nurses to report at the beginning of each period such that the total number of nurses who have to report to duty in a day is minimized.

Internal number	Time Period		Minimum number of nurses required
	From	To	
1	12 midnight	4.00a.m	20
2	4.00a.m.	8.00a.m	25
3	8.00a.m	12Noon	35
4	12 Noon	4.00p.m	32
5	4.00p.m	8.00p.m	22
6	8.00p.m.	12midnight	15

(4 Marks)

c) Consider the following Linear Programming Model and solve it using the two-phase Method

Minimize $z = 12x_1 + 18x_2 + 15x_3$

Subject to: $4x_1 + 8x_2 + 6x_3 \geq 64$

$3x_1 + 9x_2 \geq 15$

$6x_1 + 6x_2 + 12x_3 \geq 96$

$x_1 \quad x_2 \quad and \quad x_3 \geq 0$

(8 Marks)

QUESTION FIVE (20 MKS)

A dairy farm has three plants located throughout a city. Daily mild production at each plant is as follows.

Plant 1 - 8 million litres

Plant 2 - 17 million litres

Plant 3 - 5 million litres

Each day the farm must fulfil the needs of four distribution centres. Minimum requirement at each centre is as follows.

Distribution centre 1 - 7 million litres.

Distribution centre 2 - 9 million litres.

Distribution centre 3 - 10 million litres.

Distribution centre 4 - 4 million litres.

The cost of shipping one million litres of milk form each plant to each distribution center is given in the following table in hundreds of shillings.

	Distribution centres			
plants	1	2	3	4
1	2	3	11	7
2	1	2	6	1
3	5	8	15	9

The dairy farm wishes to decide as to how much should be the shipment from which plant to which plant to which distribution centre so that the cost of shipment may be minimum.

(i) Formulate the transportation matrix (4 marks)

(ii) Obtain the initial feasible solution using the following methods. Northwest corner cell method, Least cost cell method and Vogel's Approximation Method. (12 marks)

(iii) Find the optimal solution (4 marks)