(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **MAT 304/MAA 321/MAA 225**

COURSE TITLE: **COMPLEX ANALYSIS I**

DATE: 19/01/2022 **TIME:** 11 AM -1 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE COMPULSORY (30 MARKS)

a) Define the following terms

(i) Simply connected region (2 mks)

(i) Multiply connected region (2 mks)

b) Evaluate $\lim_{z \to \frac{i}{4}} \frac{(3z-2)(z+i)}{(iz-1)^2}$ (2 mks)

c) Given $z = 1 - 3i$ determine the modulus and argument of z (3 mks)

d) Using De Moivre's theorem show that

$sin3\theta = 3cos^2\theta sin\theta - sin^3\theta$ (4 mks)

e) Show that for the complex variable z, $\sin(z_1 + z_2) = sinz_1cosz_2 + cosz_1sinz_2$

(5 mks)

f) Find a function $U(x,y)$ such that;

$f(z) = U(x,y) + iV(x,y)$, given that $V(x,y) = 4x^2y - \frac{y}{x^2+y^2}$ (6 mks)

g) Find the residuals of $f(z) = \frac{z^2-2z}{(z+1)^2(z^2+4)}$ at all its poles and hence evaluate

$\oint_C f(z)dz$ (6 mks)

QUESTION TWO (20 MARKS)

a) Given that $w = f(z) = z(3 - 2z)$. Find the values of w corresponding to

$z = 2 - i$ (5 mks)

b) Find the analytic function $w = f(z)$ if its imaginary part is

$V(x,y) = 2xy + 3x$ and if $f(-i) = 2$ (5mks)

c) Evaluate $\int_{1+i}^{3+2i} (x^2 - 2ixy)dz$ (5 mks)

d) State and prove Cauchy Riemann equations (5 maks)

QUESTION THREE (20 MARKS)

a) Using Cauchy's integral formula, evaluate $\int_C \frac{z+1}{z^3-9z}dz$ where C is $|z-3| = 2.5$

(7 marks)

b) Prove that $\oint zdz = 0$ (6 mks)

c) Using residue theorem, evaluate $I = \oint_C \frac{z^2}{(z-1)^2(z-2)} dz$, where C is $|z| = 3$ (7 marks)

QUESTION FOUR (20 MARKS)

a) Consider the function $f(z) = 6x + 2y + (-x + 5y)i$, show that the function $f(z)$ is not differentiable (10 marks)

b) Evaluate $\int_0^{1+2i} (2x + y - ix^2)\, dz$ along the imaginary axis from

z = 0 to z = 2i and then along a line parallel to the real axis from z =2 i to z = 1 +2 i

(10 marks)

QUESTION FIVE (20 MARKS)

a) Find the first four terms of the Taylor series expansion of $f(z) = \ln(3 + z)$ about the point $z = 0$ (10 marks)

b) Locate and name the singularities in the finite Z-plane $f(z) = \frac{z}{(z^2+9)^2}$ and determine whether it is isolated singularity or not . (10 marks)