



(Knowledge for Development)

KIBABII UNIVERSITY
UNIVERSITY EXAMINATIONS
2020/2021 ACADEMIC YEAR
FIRST YEAR FIRST SEMESTER
SPECIAL/SUPPLEMENTARY EXAMINATION
FOR THE DEGREE OF MASTER OF SCIENCE IN PURE AND
APPLIED MATHEMATICS

COURSE CODE: MAT 817/869

COURSE TITLE: COMPLEX ANALYSIS I

DATE: 20/01/2022 TIME: 8 AM -11 AM

INSTRUCTIONS TO CANDIDATES

Answer Any THREE Questions

TIME: 3 Hours

QUESTION ONE [20 MARKS]

Consider the triangle $P(0, 0)$, $Q(2, 0)$ and $R(2, 4)$

- (i) Draw the triangle and its image under $T(z) = 2z^2 + (3 - 2i)$ (12 mks)
 (ii) Discuss conformity of T at $P(0,0)$ and $R(2,4)$ (8 mks)

QUESTION TWO [20 MARKS]

(a) Find the Laurent series about the indicated singularity for the function

$$f(z) = \frac{1}{(z+1)(z+3)} \quad z > 3 \quad (4 \text{ mks})$$

(b) Evaluate $\oint_C \frac{2z^2+z}{z^2-1} dz$ where C is a circle $|z - 1| = 1$ (5 mks)

(c) Evaluate $(3x + y)dx + (2y - x)dy$

- (i) Along the curve $y = x^2 + 1$ (5 mks)
 (ii) Straight lines from $(0,1)$ to $(0,5)$ and then $(0,5)$ to $(2,5)$ (6 mks)

QUESTION THREE [20 MARKS]

(a) Show that $\sec^{-1}(z) = \frac{1}{i} \ln \left(\frac{1 + \sqrt{1-z^2}}{z} \right)$ (5 mks)

(b) Find the residuals of the function $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ (5 mks)

(c) Evaluate $\oint_C (7x - 2y + 7)dx + (3x - 4y - 10)dy$ around a triangle in the xy plane with vertices at $(0,0)$, $(2,0)$ and $(2,3)$ (5 mks)

(d) Determine the number of zeros of $z^6 + 5z^2 - z = 1$ interior to $|z| = 1$ (5 mks)

QUESTION FOUR [20 MARKS]

(a) Prove that the function $f_1(z) = \int_0^\infty t^3 e^{-zt} dt$ is analytic at all points of z for which $\text{Re}z > 0$ (6 mks)

(b) State and prove the Rouché's theorem (14 mks)

QUESTION FIVE [20 MARKS]

(a) State the following terms (2 mks)

- (i) Analytic continuation (2 mks)
 (ii) Conformal mapping (2 mks)

(b) If $f(z) = z^5 - 2z^3 + 3z + 2 - i$, evaluate

$$\int_C \frac{f'(z)}{f(z)} dz \text{ where } C \text{ encloses all zeros of } f(z) \quad (4 \text{ mks})$$

(c) Determine the linear fractional transformation that maps $z = 0, -i, -1$ onto $w = i, 1, 0$ respectively (6 mks)

(d) Evaluate $\oint_C (z - \text{Re}(z))dz$ $C: |z| = 2$ (6 mks)