(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF MASTER OF SCIENCE IN PURE AND APPLIED MATHEMATICS

COURSE CODE: **MAT 817/869**

COURSE TITLE: COMPLEX ANALYSIS I

DATE: 20/01/2022 **TIME:** 8 AM -11 AM

INSTRUCTIONS TO CANDIDATES

Answer Any THREE Questions

TIME: 3 Hours

QUESTION ONE [20 MARKS]

Consider the triangle $P(0,0)$, $Q(2,0)$ and $R(2,4)$

(i) Draw the triangle and its image under $T(z) = 2z^2 + (3-2i)$ (12 mks)

(ii) Discuss conformity of T at $P(0,0)$ and $R(2,4)$ (8 mks)

QUESTION TWO [20 MARKS]

(a) Find the Laurent series about the indicated singularity for the function

$f(z) = \frac{1}{(z+1)(z+3)} \quad z > 3$ (4 mks)

(b) Evaluate $\oint_C \frac{2z^2+z}{z^2-1}dz$ where C is a circle $|z-1| = 1$ (5 mks)

(c) Evaluate $(3x+y)dx + (2y-x)dy$

(i) Along the curve $y = x^2 + 1$ (5 mks)

(ii) Straight lines from (0,1) to (0,5) and then (0,5) to (2,5) (6 mks)

QUESTION THREE [20 MARKS]

(a) Show that $sec^{-1}(z) = \frac{1}{i} ln\left(\frac{1+\sqrt{1-z^2}}{z}\right)$ (5 mks)

(b) Find the residuals of the function $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ (5 mks)

(c) Evaluate $\oint_C (7x-2y+7)dx + (3x-4y-10)dy$ around a triangle in the xy plane with vertices at (0,0), (2,0) and (2,3) (5 mks)

(d) Determine the number of zeros of $z^6 + 5z^2 - z = 1$ interior to $|z| = 1$ (5 mks)

QUESTION FOUR [20 MARKS]

(a) Prove that the function $f_1(z) = \int_0^\infty t^3 e^{-zt} dt$ is analytic at all points of z for which $Rez > 0$ (6 mks)

(b) State and prove the Rouche's theorem (14 mks)

QUESTION FIVE [20 MARKS]

(a) State the following terms

(i) Analytic continuation (2 mks)

(ii) Conformal mapping (2 mks)

(b) If $f(z) = z^5 - 2z^3 + 3z + 2 - i$, evaluate $\int_C \frac{f^l(z)}{f(z)}dz$ where C encloses all zeros of $f(z)$ (4 mks)

(c) Determine the linear fractional transformation that maps $z = 0, -i, -1$ onto $w = i, 1, 0$ respectively (6 mks)

(d) Evaluate $\oint_C (z - Re(z))dz \quad C: |z| = 2$ (6 mks)