KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS
2020/2021 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER
SPECIAL/SUPP EXAMINATIONS

FOR THE DEGREE OF B.ED (SCIENCE)

COURSE CODE: SPH 222

COURSE TITLE: QUANTUM MECHANICS

DURATION: 2HRS

DATE: 20/1/2022 **TIME:** 8-10AM

INSTRUCTIONS TO CANDIDATES

- Answer question ONE (compulsory) and any TWO of the remaining questions.
- Attempted questions must be indicated on front cover of answer booklet.
- Every question should be started on new page and question indicated respectively.
- The symbols used bears the usual meaning.

Question One

a. Define the Schwartz inequality. (2mks)

b. Discuss any four postulates of quantum mechanics (4mks)

c. Define the Eigen values and Eigen function. (2mks)

d. Using the uncertainty principle for position and momentum, estimate the ground state energy for an infinite well of width a. (4mks)

e. Commute the following operators $[x, P_y]$ *and* $[z, p_x]$. (4mks)

f. Proof that for generalized form of uncertainty principle for any two non-compatible observables A and B is given by

$\sigma(A)\sigma(B) =\geq \frac{1}{2}|\langle[A,B]\rangle|$ (5mks)

g. State the Jacobi identity on properties of operators (2mks)

h. Comment on the statement that in the photoelectric effect the photon transfers all of its energy while in the Compton effect only part of the energy is transferred to the electron. (3mks)

i. The inadequacy in classical mechanics led to the need for quantum mechanics. State and discuss failure of classical mechanics that resulted to the birth of quantum mechanics. (4mks)

Question Two

a. A particle of mass m is confined to an infinite potential well, $V(x) = 0$ for $-\frac{L}{2} \leq x \leq \frac{L}{2}$ and $V(x) = \infty$ otherwise. The wave functon at time $t = 0$ is $\Psi(x, 0) = \frac{1}{\sqrt{2}}[\Psi_1(x) + \Psi_2(x)]$ where $\Psi_1(x) = \sqrt{\frac{2}{L}}\cos(\frac{\pi x}{L})$ with energy $E_1 = \frac{\hbar^2\pi^2}{2mL^2}$ and $\Psi_2(x) = \sqrt{\frac{2}{L}} sin\frac{2\pi x}{L}$ with energy $E_2 = 4E_1$.

i. Write down the equation for wave function $\Psi(x,t)$ at all subsequent times. (2mks)

ii. Derive an expression for the probability density $P(x,t)$. (10mks)

b. A particle of mass m moves non-relativistically in one dimension in a potential given by $V(x) = a\delta(x)$, where $\delta(\mathrm{x})$ is the usual Dirac delta function. The particle is bound. Find the value of x_0 such that the probability of finding the particle with $|x| < x_0$ is exactly equal to $\frac{1}{2}$. (8mks)

Question Three

a. A beam of particles of mass m and energy E is incident from the left on a step potential given by $V = 0$ for $x < 0$ and $V = V_0$ for $x > 0$, where $E > V_0$. The general solutions of the Schrödinger equation valid on the two sides of the step are of the forms

$\Psi_L = Aexp\left(ikx - \frac{iEt}{\hbar}\right) + Bexp\left(-ikx - \frac{iEt}{\hbar}\right)$ and $\Psi_R = Texp\left(ik'x - \frac{iEt}{\hbar}\right)$, respectively. Show that $k = \sqrt{\frac{2mE}{\hbar^2}}$ and $k' = \sqrt{\frac{2m(E-V_0}{\hbar^2}}$

(12mks)

b. A particle is in the n^{th}energy state $\Psi_n(x)$ of an infinite square well potential with width L. Determine the probability $P_n(\frac{1}{a})$ that the particle is confined to the first $\frac{1}{a}$ of the width of the well. (8mks)

Question Four

a) The time-independent Schrodinger equation in one dimension is given as $\frac{d^2\Psi(x)}{dx^2} + \frac{2m}{\hbar^2}[E - U(x)]\Psi(x) = 0$ where E and $U(x)$ are the total and potential energies of a particles of mass m respectively. Find the solution to TISE when

i. At ground state (6mks)

ii. Excited state (6mks)

b) Using wave mechanics, show that time dependent Schrödinger equation in three dimension is given by the following equation $\frac{\hbar}{i}\frac{\partial\Psi}{\partial t} = \frac{\hbar}{2m}\nabla^2\Psi - V\Psi$ where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the laplacian operator (8mks)

Question Five

a. Consider a square well having an infinite wall at x = 0 and a wall of height U at x = L. For the case E < U, obtain solutions to the time independent Schrodinger equation (TISE) inside the well ($0 \leq x \leq L$) and in the region beyond (x > L) that satisfy the appropriate boundary conditons at x = 0 and at x = 1. (12 marks)

b. Derive the Compton equation the proofs the existence of quantum mechanics. (8mks)