(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

SPECIAL/SUPPLIMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE (MATHEMATICS)

COURSE CODE: **MAT 428**

COURSE TITLE: MATHEMATICAL MODELING

DATE: 20/01/2022 **TIME:** 8 AM -10 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

Question 1: Compulsory (30 marks)

a). Define the following terms

i). Model (2mks)

ii). Mathematical model (2mks)

b). State four steps in the mathematical modeling process (4mks)

c). Differentiate between stochastic and deterministic models (2mks)

d). A breeder reactor converts relatively stable Uranium-238 into the isotope Plutonium 239. After 15 years it is determined that 0.043% of the initial amount A_0 has disintegrated. Find the half life of this isotope if the rate of disintegration is proportional to the amount remaining (6mks)

e). Suppose the current Kenyan population is 40 million and the birth rate is 0.02 and the death rate is 0.01

i). What will be the population in ten years if the population keeps growing at the same rate? (3mks)

ii). How many years will it take for the population to double its initial size? (3mks)

iii). How many births will occur between $t = 10$ and $t = 11$? (2mks)

f). A body cools from 90^0C to 70^0C in 3 minutes at a surrounding temperature of 15^0C.Determine how long it will take for the body to cool to 50^0C (6mks)

Question 2 (20mks)

Discuss the continuous population models found in population dynamics. Include an explanation of all the variables and parameters used (20mks)

Question 3 (20mks)

a) Find the solution of the differential equation
$\frac{dp}{dt} = p\left(r - \frac{r}{k}p\right)$ (7mks)

b) Suppose a student carrying a flu virus returns to an isolated college campus of 1000 students. If it is assumed that the rate at which the virus spreads is proportional not only to the number x of infected students but also to the number of students not infected. Use the solution in (a) to determine the number of infected students after 6 days if its further observed that the number of infected students after 4 days is 50. What happens as t increases? (5mks)

c) The simplest epidemic model is the SIR model

$$\frac{dS}{dt} = -aSI$$

$$\frac{dI}{dt} = aSI - bI$$

$$\frac{dR}{dt} = bI$$

where $S(t)$ represents the Susceptible population, $I(t)$ the infected population and $R(t)$ the Removed population. Find the equilibrium points for the above system and study its stability (8mks)

Question 4 (20mks)

a). i). With an example explain the Predator-Prey model with differential equations (4mks)
ii). Describe the dynamics of the above system when the prey has unlimited and limited resources (6mks)

b) i). An electrical circuit contains inductance L and resistance R connected to a constant voltage source E. The current i is given by the differential equation

$$E - L\frac{di}{dt} = Ri$$

where L and R are constants. Find the current in terms of t given that when $t = 0, i = 0$ (5mks)

ii). A 12 volt battery is connected to a series circuit in which the inductance is $\frac{1}{2}$ henry and the resistance is 10 ohms. Determine the current i if the initial current is 0 (5mks)

Question 5 (20mks)

a). An enzymatic reactions involves a substrate S reacting with an enzyme E to form a complex SE which in turn is converted into product P as shown below

$S + E \underset{k_{-1}}{\overset{k}{\longrightarrow}} SE \quad , SE \overset{k_2}{\longrightarrow} P + E$ The enzyme is then free to participate in another reaction. Develop a model for the above reactions (5mks)

b) Find the displacement of the spring after time t described by the system of differential equations

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{r}{m} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{m} \end{pmatrix} f(t)$$

in the case where the mass on the string is 1, the damping constant $r = 5$, the spring constant $k = 4$, the forcing function $f(t) = t$ and the initial extension of the spring is 0 with 0 initial velocity (10mks)

c) A new type of flu known as elephant flue has been identified by KEMRI. It is highly infectious and transmitted very easily through the air in the rainy season. The whole population is able to catch the disease and once a healthy person has become infected it takes a couple of days before they themselves get ill and can transmit the disease. Most people recover naturally but a small proportion dies from the disease. Once someone has recovered they remain immune for the period of the outbreak. When an outbreak occurs it is expected to only last a few months. Write down your proposed model including an explanation of all the variables and parameters used in the model (5mks)