(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

SPECIAL/SUPPLIMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: MAT 421

COURSE TITLE: PDE I

DATE: 14/01/2022 **TIME:** 8:00 AM - 10:00 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

(a) Define a pfaffian differential form and a pfaffian differential equation (2 marks)

(b) Form partial differential equations from

(i) $z = ax + by + a^2 + b^2$ (5 marks)

(ii) $(x-2)^2 + (y-b)^2 + z^2 = 25$ (5 marks)

(iii) $z = a\log\left[\frac{b(y-1)}{1-x}\right]$ (5 marks)

By eliminating the arbitrary constants

(c) Obtain the solution of the first order linear partial differential equation (8 marks)

$$y\frac{\partial u}{\partial x} - x\frac{\partial u}{\partial y} + xu = 0$$

Satisfying the condition u=y when $x^2 + 2y^2 = 4$

(d) Find the equation of the

(i) Tangent plane

(ii) Normal line to the surface $x = u, y = v, z = u^2 + v^2$ at the point $p_0(1,-1,2)$ (5marks)

QUESTION TWO (20 MARKS)

Solve the following differential equations

(a) $y^2zp + x^2zq = y^2x$ (5marks)

(b) $p + 3q = 5z + tan(y - 3x)$ (5 marks)

(c) $(y-z)p + (x-y)q = z - x$ (5marks)

(d) $y^2p - xyz = x(z - 2y)$ (5marks)

QUESTION THREE (20 MARKS)

(a) Using the fact coefficients of the differential equation $(y^2 + z^2)dx + xydy + xzdz = 0$ are homogeneous functions in x, y and z of the same degree obtain the general solution.(6marks)

(b) Show that the first order partial differential equation $xp - yq = x, x^2p + q = xz$, are compatible (6marks)

(c) Solve the equations

(i) $p^2 + q^2 = npq$ (4 MKS)

(4marks)

(ii) $1 + p^2 = qz$

QUESTION FOUR (20 MARKS)

(a) Solve the equations

(i) $z^2(p^2 + q^2) = x^2 + y^2$ [6 marks]

(ii) $\frac{p}{x^2} + \frac{q}{y^2} = z$ [5 marks]

(b) Consider the Lagranges partial differential equation

$x(y - z)p + y(z - x)q = z(x - y)dz$ where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$ write the corresponding Lagranges auxiliary equations and hence find its integral surface that passes through the curve $z = 1, y = x$

(9 marks)

QUESTION FIVE (20 MARKS)

(a) Solve the equation

$2x^4p^2 - yzp - 3z^2 = 0$ [10marks]

(b) Find the equation of the tangent plane to the hyperboloid $4x^2 - 9y^2 - 9z^2 - 36 = 0$ at the point (3,2,2) (10 marks)