(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

SPECIAL/SUPPLIMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **MAT 406**

COURSE TITLE: FIELD THEORY

DATE: 17/01/2022 **TIME:** 11 AM -1:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

Question 1 (30 marks)

a) Define the following terms

i) Zero divisor (3 marks)

ii) Integral domain (2 marks)

iii) Division ring (2 marks)

iv) Field (2 marks)

v) Homomorphism (3 marks)

b) If R is an integral domain and $p(x)$ and $q(x)$ are nonzero elements of $R(x)$, then $\deg(p(x)+q(x)) = \deg p(x) + \deg q(x)$. Prove (6 marks)

c) $R(x)$ is an integral domain iff R is an integral domain. Prove (6 marks)

d) Given that $f(x) = 3x^2 + 2$

$g(x) = 4x^4 + 2x^3 + 6x^2 + 4x + 5$ in $Z_7[x]$. Find $q(x)$ and $r(x)$ such that $g(x) = f(x).q(x) + r(x)$. (6marks)

Question 2 (20 marks)

a) Let F be an arbitrary field and $f(x)$ be a polynomial over F. Define a zero of $f(x)$.

(3 marks)

b) If $f(x)$ is a polynomial over F and $c \in F$. The remainder in the division of $f(x)$ by $x - c$ is $f(c)$. Prove (5 marks)

c) A polynomial $f(x)$ over F has a factor $x - c$ in $F(x)$ iff $c \in F$ is a zero of $f(x)$. Prove

(3 marks)

d) Let F be a field then $f(x)$ is a unit in $F(x)$ iff $f(x)$ is a nonzero constant polynomial. Prove. (9 marks)

Question three (20 marks)

a) Determine whether the following are irreducible over Z_5

i) $f(x) = x^3 + 2x^2 - 3x + 4$ (5 marks)

ii) $g(x) = x^2 + 3x + 4$ (5 marks)

b) Express $x^4 - 4$ as a product of irreducible factors in

i) $Q[x]$ (3 marks)

ii) $R[x]$ (3 marks)

iii) ¢$[x]$ (4 marks)

Question four (20 marks)

a) Define the following:

i) Characteristic of a field F (3 marks)

ii) Field extensions (3 marks)

iii) The degree of a field extension (3 marks)

iv) Let α be algebraic over a field F. Define the minimal polynomial for α over F. (3 marks)

b) The characteristic of a field F, Ch(F) is either zero or a prime p. Prove (8 marks)

Question five (20 marks)

a) The algebraic element α is algebraic over F iff the simple extension $F(\alpha)/F$ is finite.

Prove. (12 marks)

b) $F(\alpha, \beta) = (F(\alpha))(\beta)$. Prove (8 marks)