(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

SECOND YEAR SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: MAA 211 / MAT 203

COURSE TITLE: VECTOR ANALYSIS

DATE: FRI 15/07/2022 TIME: 8:00 AM – 10:00 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS): COMPULSORY

a) Define the terms: (2 marks)

(i) Scalar product (2 marks)

(ii) Vector product

b) Find the angle between the vectors $\vec{A} = 2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$ and $\vec{B} = 6\underset{\sim}{i} - 3\underset{\sim}{j} + 2\underset{\sim}{k}$ (6 marks)

c) Given $\phi = 2x^3 y^2 z^4$, find $\nabla \bullet \nabla\phi$ (4 marks)

d) Find the work done in moving an object along a straight line from $(3,2,-1)$ to $(2,-1,4)$ in a force field given by $\vec{F} = 4\underset{\sim}{i} - 3\underset{\sim}{j} + 2\underset{\sim}{k}$ (5 marks)

e) Determine the value "a" so that $\vec{A} = 2\underset{\sim}{i} + a\underset{\sim}{j} + \underset{\sim}{k}$ and $\vec{B} = 4\underset{\sim}{i} - 2\underset{\sim}{j} - 2\underset{\sim}{k}$ are orthogonal (5 marks)

f) Show that the linearly independent solutions of $y'' - 2y' + 2y = 0$ are $e^x \sin x$ And $e^x \cos x$ (6 marks)

QUESTION TWO (20 MARKS)

a) Find $\nabla\phi$ if $\phi = \log\left|\vec{r}\right|$ (6 marks)

b) If $\vec{A} = 5t^2\underset{\sim}{i} + t\underset{\sim}{j} - t^3\underset{\sim}{k}$ and $\vec{B} = \sin t\,\underset{\sim}{i} - \cos t\,\underset{\sim}{j}$, find:

(i) $\dfrac{d}{dt}\left(\vec{A} \bullet \vec{B}\right)$ (4 marks)

(ii) $\dfrac{d}{dt}\left(\vec{A} \times \vec{B}\right)$ (4 marks)

c) Find a unit tangent vector on the curve $x = t^2 + 1$, $y = 4t - 3$, $z = 2t^2 - 6t$ (6 marks)

QUESTION THREE (20 MARKS)

a) If $\vec{A} = xz^3\underset{\sim}{i} - 2x^2yz\underset{\sim}{j} + 2yz^4\underset{\sim}{k}$, find $\nabla \times \vec{A}$ at point $(1,-1,1)$ (6 marks)

b) Determine a unit vector perpendicular to the plane of $\vec{A} = 2\underset{\sim}{i} - 6\underset{\sim}{j} - 3\underset{\sim}{k}$ and

$\vec{B} = 4\underset{\sim}{i} + 3\underset{\sim}{j} - \underset{\sim}{k}$ (5 marks)

c) A particle moves along a curve whose parametric equations are $x = e^{-t}$, $y = 2\cos 3t$ and $z = 2\sin 3t$ where t is the time. Determine:

(i) Velocity and acceleration at time t (6 marks)

(ii) Magnitude of the velocity and acceleration at $t = 0$ (3 marks)

QUESTION FOUR (20 MARKS)

a) Prove that $\nabla(F + G) = \nabla F + \nabla G$, where F and G are differential scalar functions of x, y and z (5 marks)

b) Evaluate: $\left(2\underset{\sim}{i} - 3\underset{\sim}{j}\right)\left[\left(\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}\right) \times \left(2\underset{\sim}{i} - \underset{\sim}{k}\right)\right]$ (3 marks)

c) Show that $\vec{F} = \left(2xy + z^3\right)\underset{\sim}{i} + x^2\underset{\sim}{j} + 3xz^2\underset{\sim}{k}$ is:

(i) A conservative force field (4 marks)

(ii) A scalar potential (3 marks)

d) If $\vec{A} = \left(3x^2 + 6y\right)\underset{\sim}{i} - 14yz\underset{\sim}{j} + 20xz^2\underset{\sim}{k}$, evaluate $\int \vec{A} \bullet d\vec{r}$ from $(0,0,0)$ to $(1,1,1)$ along the following paths: $x = t$, $y = t^2$ and $z = t^3$ (5 marks)

QUESTION FIVE (20 MARKS)

a) If $\vec{V} = \vec{\omega} \times \vec{r}$, prove that $\vec{\omega} = \frac{1}{2} curl \vec{V}$ where $\vec{\omega}$ is a constant vector (5 marks)

b) A particle moves so that its position vector is given by $\vec{r} = \cos\omega t\,\underset{\sim}{i} + \sin\omega t\,\underset{\sim}{j}$, where ω is a constant. Show that:

(i) The velocity $\vec{V}$ of the particle is perpendicular to $\vec{r}$ (5 marks)

(ii) The acceleration $\vec{a}$ is directed towards the origin and has a magnitude proportional to the distance from the origin (5 marks)

c) Prove that $\nabla \bullet \left(\phi \vec{A}\right) = (\nabla\phi) \bullet \vec{A} + \phi\left(\nabla \bullet \vec{A}\right)$, where ϕ is a function of (x, y, z) (5 marks)

END