KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUATION SCIENCE

COURSE CODE: MAP 112

COURSE TITLE: BASIC MATHEMATICS AND ANALYTIC GEOMETRY

DATE: 19/07/2022   **TIME:** 08:00 AM- 10:00 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) Define the following terms
   i.   Ellipse
   ii.   Hyperbola   (4 marks)

b) Analyze the equation $(y - 2)^2 = 8(x + 3)$ (5 marks)

c) Convert the following equation to polar $(x - a)^2 + (y - b)^2 = a^2 + b^2$ (4 marks)

d) Convert the cartesian equations given below to polar equation.
   i. $x^2 + y^2 - 6x = 0$
   ii. $x = 2$ (5 marks)

e) Find the center, vertex, foci and asymptotes of the hyperbola $\frac{(x-1)^2}{4} - \frac{(y+2)^2}{1} = 1$ and
   sketch the graph.   (6 marks)

f) Convert the following equation to Cartesian form, giving its vertex and sketch.
   $x = t^2 + t, y = 2t - 1 -3 \leq t \leq 3$ (6 marks)

QUESTION TWO (20 MARKS)

a) Find the vertex, axis of symmetry, focus and directrix of the parabola $y^2 + 8x - 6y + 1 = 0$ (6 marks)

b) Find the coordinates of the foci, the lengths of major and minor axes and graph   (6 marks)
   $$x^2 + 4y^2 = 4$$

c) Analyze $169x^2 + 144y^2 = 24336$ (4 marks)

d) Find equation of an ellipse with foci $(-3,0)$ and $(3,0)$ that passes through $(0,4)$ (4 marks)

QUESTION THREE (20 MARKS)

a) State the factor theorem   (2 marks)
b) Solve the following using synthetic division,   (3 marks)
   $(x^3 + 8) \div (x + 2)$

c) Find the zeros of $f(x) = x^3 - 6x^2 + 11x - 6$ (4 marks)

d) Write the standard forms of the polar equations of conics.   (4 marks)

e) Verify whether the circles $x^2 + y^2 - 6x + 8y - 23 = 0$ and $x^2 + y^2 - 5x - 2y + 16 = 0$ are orthogonal. (4 marks)

f) Analyze the graph of $r = \frac{4}{2+\cos\theta}$ (3 marks)

QUESTION FOUR (20 MARKS)

a) Find the equation of a parabola, directrix and its sketch having its vertex at $(-1, 2)$ and focus at $(0,2)$. (5 marks)

b) Write an equation for a parabola that opens to the left with vertex $(0,2)$ and passes through $(-6,-4)$ (5 marks)

c) Sketch and derive the equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (7 marks)

d) Using trigonometric substitution convert the following equation into parametric form.
$$\frac{x^2}{4} - \frac{y^2}{9} = 1$$
(3 marks)

QUESTION FIVE (20 MARKS)

a) Find the point and angle of intersection of the lines $5x - 6y - 1 = 0$ and $3x + 2y + 5 = 0$ (4 marks)

b) Convert the polar equation given below to Cartesian equations and identify their graphs.
   i. $r = 2a \cos\theta$
   ii. $r = 2a \cos\theta + 2b \sin\theta$ (6 marks)

c) Convert the following set of parametric equations to rectangular equations.
   i. $x = t^2 + t, y = 2t - 1$
   ii. $x = 5\cos t, y = 2\sin t, \ 0 \le \theta \le 2\pi$. (6 marks)

d) Convert the following polar coordinates to rectangular coordinates
   i. $(2, \frac{\pi}{2})$
   ii. $(-1, 0)$ (4 marks)