(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION

MATHEMATICS

COURSE CODE: MAA 121/MAT 102

COURSE TITLE: FOUNDATION MATHEMATICS II

DATE: 29/07/22 **TIME:** 11.00 AM -1.00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of **3 Printed Pages**. Please Turn Over.

QUESTION ONE (30 MARKS)

(a) Using examples define (4 mks)

(i) A matrix

(ii) A vector

(b) Find B if $(2I + B^T)^{-1} = \begin{bmatrix} -3 & 1 \\ -2 & 1 \end{bmatrix}$ (5 mks)

(c) If $\boldsymbol{a} = i - 5K$ and $\boldsymbol{b} = 2i - 3j + 2k$ evaluate $\boldsymbol{b}.(\boldsymbol{a} \times 2\boldsymbol{b})$ (5 mks)

(d) Find the angle between two vectors $2i - 3j - 5k$ and $-i + 4j - k$ (6 mks)

(e) Given that $A = \begin{bmatrix} -8 & -6 & -12 \\ 0 & 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & 6 \\ 4 & -1 \\ 7 & 0 \end{bmatrix}$ find $(A - B^T)^T$ (3 mks)

(f) Find the solution of the following system of linear equations using augmented matrices (7 mks)

$$3x + 4y + z = 1$$

$$4x + 3y - Z =- 2$$

$$2x + 3y = 0$$

QUESTION TWO (20 MARKS)

(a) Use Cramer's rule to find $x_1, x_2,$ and $x_3,$ (10 mks)

$$x_1 - 2x_2 + 3x_3 = 9$$

$$-x_1 + 3x_2 + 4 = 0$$

$$2x_1 - 5x_2 + 5x_3 = 17$$

(b) Compute the rank of $\begin{bmatrix} 1 & 1 & -1 & 4 \\ 2 & 1 & 3 & 0 \\ 0 & 1 & -5 & 8 \end{bmatrix}$ (6 mks)

(c) Given $A = \begin{bmatrix} 2b & -2b \\ 4 & -b \end{bmatrix}$ has determinant of 8 find b (4 mks)

QUESTION THREE (20 MARKS)

(a) Using the inversion algorithm find the inverse of the matrix $\begin{bmatrix} 2 & 7 & 1 \\ 1 & 4 & -1 \\ 1 & 3 & 0 \end{bmatrix}$ (10 mks)

(b) Compute the adjoint of P given

$$\begin{bmatrix} 10 & 4 & -3 \\ 0 & -2 & 5 \\ 2 & 0 & -7 \end{bmatrix}$$ (10 mks)

QUESTION FOUR (20 MARKS)

(a) Find the projection of $i - 2j + 7k$ on $-4i + 6j - 3k$ (4 mks)

(b) Show that $||a \times b|| = ||a|| ||b|| sin\theta$ (5 mks)

(c) Given that $A = \begin{bmatrix} 6 & -2 \\ 4 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} -9 & -2 \\ 4 & 12 \end{bmatrix}$
Prove that $\det(AB) = \det A \det B$ (5 mks)

(d) Reduce the system into row-echelon form hence by backward substitution solve it

$$x - 2y + 2z = 3$$ (6 mks)
$$2x + y + z = 0$$
$$x + z = -2$$

QUESTION FIVE (20 MARKS)

(a) Given $a = \langle 6,1,-2 \rangle$ and $b = \langle 4,-3,1 \rangle$ compute
 (i) $a \times b$ (3 mks)
 (ii) $b \times -3a$ (4 mks)

(b) If $detA = 10$ and $\det B = -9$ calculate $det(A^2 B^{-1} A^T B^3)$ (5 mks)

(c) Compute the determinant of $\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -6 & 2 & 3 \end{bmatrix}$ (5 mks)

(d) Determine if the two vectors are parallel, orthogonal or neither
$-i - 4j - 3k$ and $2i + 3j - k$ (3 mks)