(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2021/2022 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE MATHEMATICS

COURSE CODE: **MAP 223**

COURSE TITLE: **ALGEBRAIC STRUCTURES II**

DATE: 26/07/2022 **TIME:** 11:00 AM - 1:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE COMPULSORY (30 MARKS)

a) Define the following terms
   i. Rings (5marks)
   ii. Group (4marks)
   iii. Subgroup (2marks)

b) List all elements of S_3 in cycle notation. What is the order of each? Verify Lagrange's Theorem for S_3. (15marks)

c) Let n > 0 and k > 0. If q is the quotient and r is the remainder when m is divided by n, then q is the quotient and kr is the remainder when km is divided by kn. (4marks)

QUESTION TWO (20 MARKS)

a) Find a four-element abelian subgroup of $S5$. Write its table. (6marks)

b) Let G be a cyclic group of order 9. How many of its elements generate G? (5marks)

a) Prove that if $ab = ba$, then $(ab)^n = a^n b^n$ by induction (5marks)

b) Let G be a group and let g ∈ G prove that the cyclic subgroup $< g >$ generated by g is a subgroup of G. (4marks)

QUESTION THREE (20 MARKS)

a) Let G be the subset of $S4$ consisting of the permutations

$$e = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} \quad f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$$

$$g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix} \quad h = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}$$

Show that G is a group of permutations, and write its table (8marks)

b) Construct the cayley composition table of the Dihedral group D_3 (5marks)

c) State the Division algorithm (2marks)

d) Given the groups G and H, prove that the direct product $G \times H$ is a group. (5marks)

QUESTION FOUR (20 MARKS)

d) Define the following terms
   i. Integral domain (2marks)
   ii. Commutative ring (2marks)
   iii. Field (3marks)

e) Prove that if $abc = e$ then $cab = e$ and $bca = e$ (4marks)

a) If G is a group and a,b are elements of G then prove that
   i. $ab = ac \; implies \; b = c$ (2marks)
   ii. And $ba = ca \; implies \; b = c$ (2marks)

b) Prove that the subgroups of S_3 satisfy the lagranges theorem (5marks)

QUESTION FIVE (20 MARKS)

a) Define the following terms

  i. Dihedral Group (2marks)

  ii. Symmetric group (2marks)

b) Our checkerboard has only four squares, numbered 1, 2, 3, and 4. There is a single checker on the board, and it has four possible moves:

V: Move vertically; that is, move from 1 to 3, or from 3 to 1, or from 2 to 4, or from 4 to 2.

H: Move horizontally; that is, move from 1 to 2 or vice versa, or from 3 to 4 or vice versa.

D: Move diagonally; that is, move from 2 to 3 or vice versa, or move from 1 to 4 or viceversa.

I: Stay put.

If

$G = \{V, H, D, I\}$, and * is the operation of performing two moves successively, write the table of G and prove that (G,*) forms a group

(10marks)

b) Show that for $n \geq 1$, $8^n - 3^n$ is divisible by 5 for $n \in \mathbb{N}$ by induction (6marks)